Analysis of model error: Parameter estimation and spatial analysis of variance (ANOVA)

J. Hacker (NPS)

C. Kaufman (UC Berkeley)

J. Hansen (NRL)

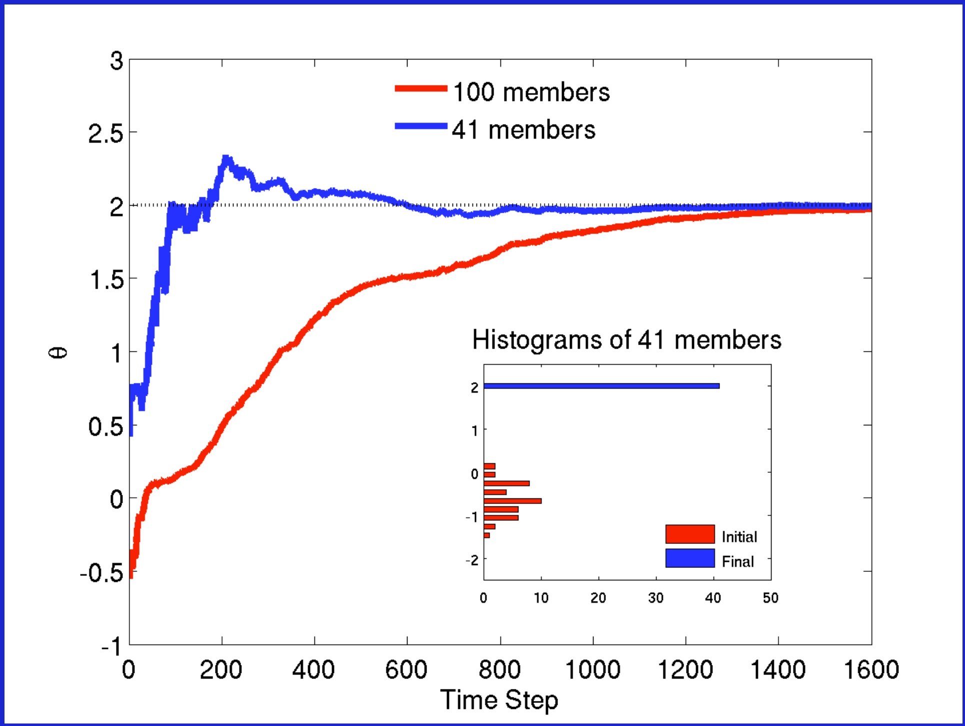
4th EnKF Workshop, April 2010

Goal

- We'd like to find an objective and flexible framework for characterizing (and ultimately fixing) model structural errors
  - Improve priors for data assimilation
  - Inform stochastic prediction schemes

This is easy…

- Lorenz 95 (40-variable) error in forcing *F*
- True *F*=8, model *F*=6
- *q* is free error parameter on RHS (a tendency on all X_i).



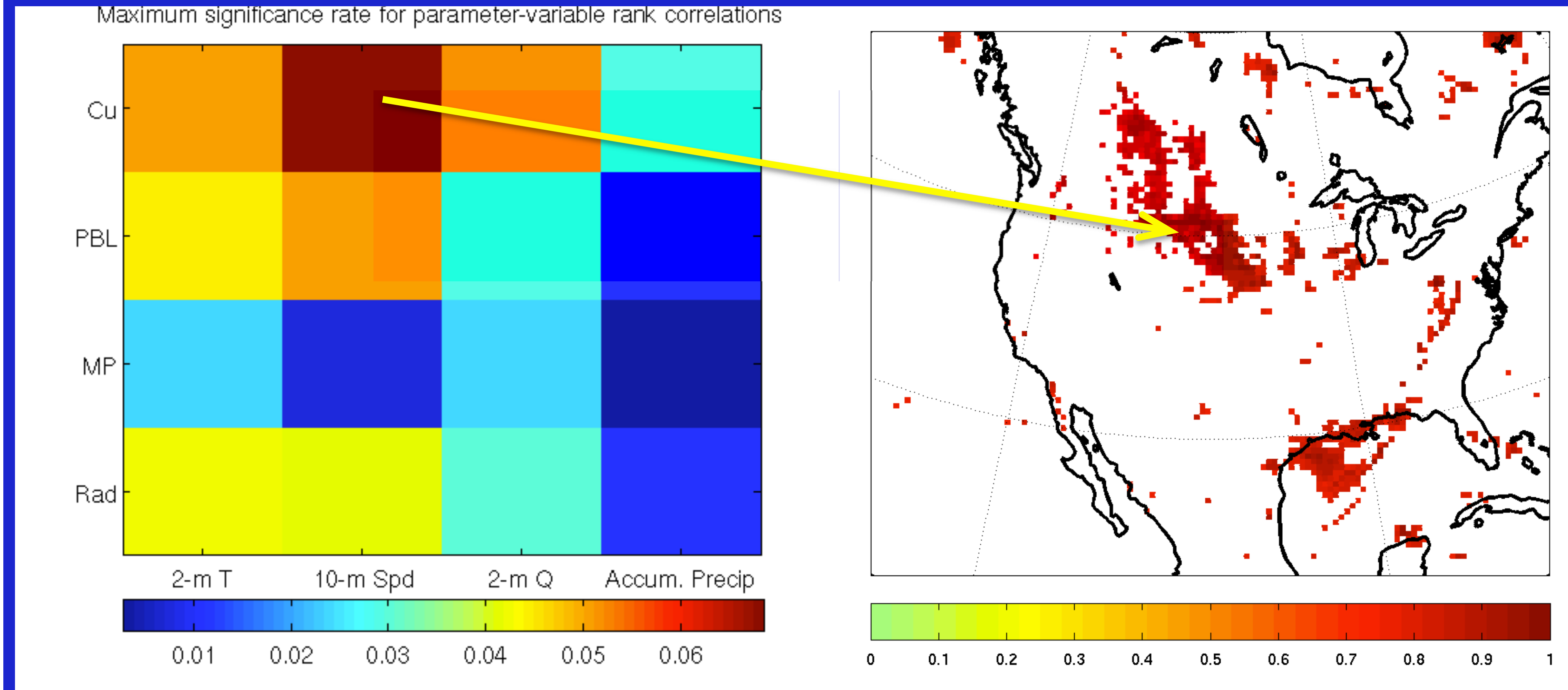
State augmentation:

$$\mathbf{z} = \begin{bmatrix} \mathbf{x} \\ \theta \end{bmatrix}$$

$$p(\mathbf{z},t \mid \mathbf{Y})$$

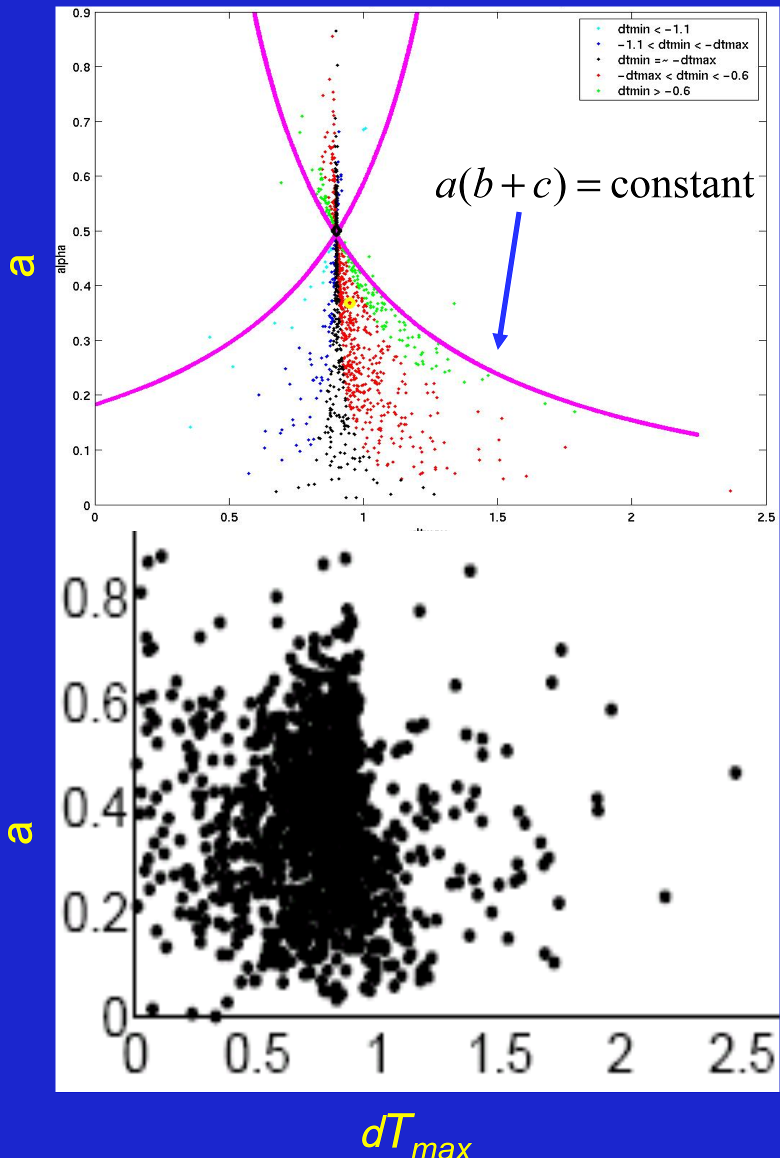
But doing this in a real model can be much more difficult.

Ensemble prediction-parameter rank correlations from 60 forecasts



Difficult to find atmospheric parameters that are strongly correlation to easily observable quantities near the surface (soil maybe easier).

Parameters appear stochastic



Cloud-base mass flux increment

$$\Delta M = \alpha(\Delta T_v + \Delta T_{sub} + \delta T_{\max}) - \lambda M_{OLD}$$

a) Distribution of parameters in the NOGAPS convective parameterization that control the amount of moist air drawn through the base of a convective cloud. Solid curves show theoretical relationship curves, points are optimized parameter values. Different colors indicate different convective regimes. Model is perfect.

b) As for a), but now with an imperfect model where the cloud-based mass flux parameters are being used to offset error in the convective momentum transport. It is possible to degrade the fidelity of the model in order to improve its output (from MIT PhD thesis of V. Khade).

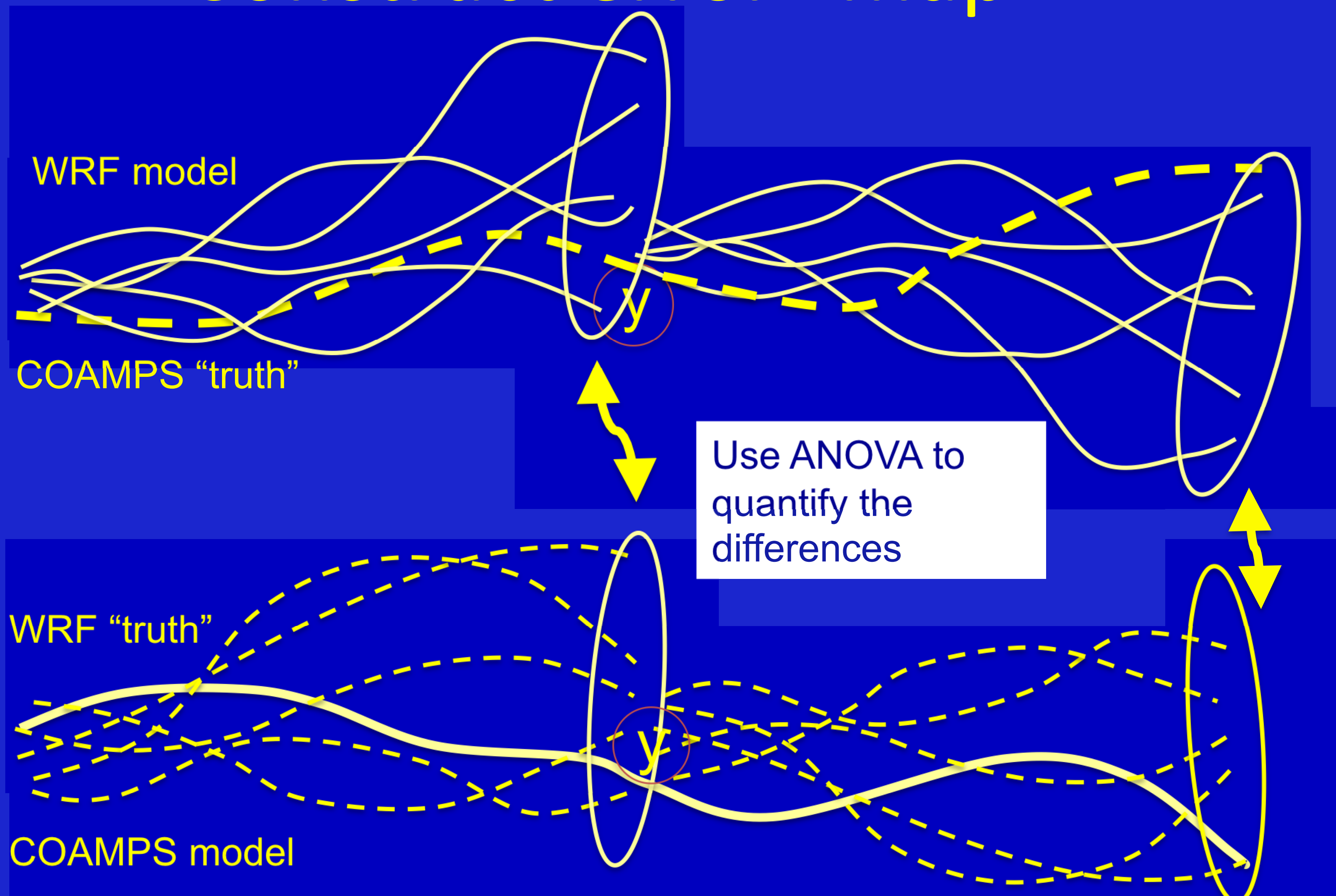
Some recent success…

- Danforth and Kalnay (2008): low-dimensional parameterization for tendency errors from forecast errors.
- Posselt and Vukecevic 2010: transformations and physical bounds helpful for estimating parameters.
- Nielsen-Gammon et al. (2010): linear and distinguishable parameters in the PBL.

Approach

1. Do the best job possible to produce a "map" from one model to another. This is what we'll try to recover.
2. Find a basis that is compatible with the map with estimable coefficients (parameters). Ensure that it is not restrictive.
3. Estimate the parameters and make sure that the parameters are capturing the correct errors.
4. Test error estimates using EnKF assimilation.

Construct error "map"



ANOVA Models

- Analysis of variance (ANOVA) models are a classical statistical technique for studying whether a scalar outcome differs according to some categorical variable or *factor*. The classic example is a clinical trial, where the factor is drug/no drug.

- The main idea of the model is that we *batch* certain sets of observations according to their expected values and then say that they have a common (conditional) distribution. For example,

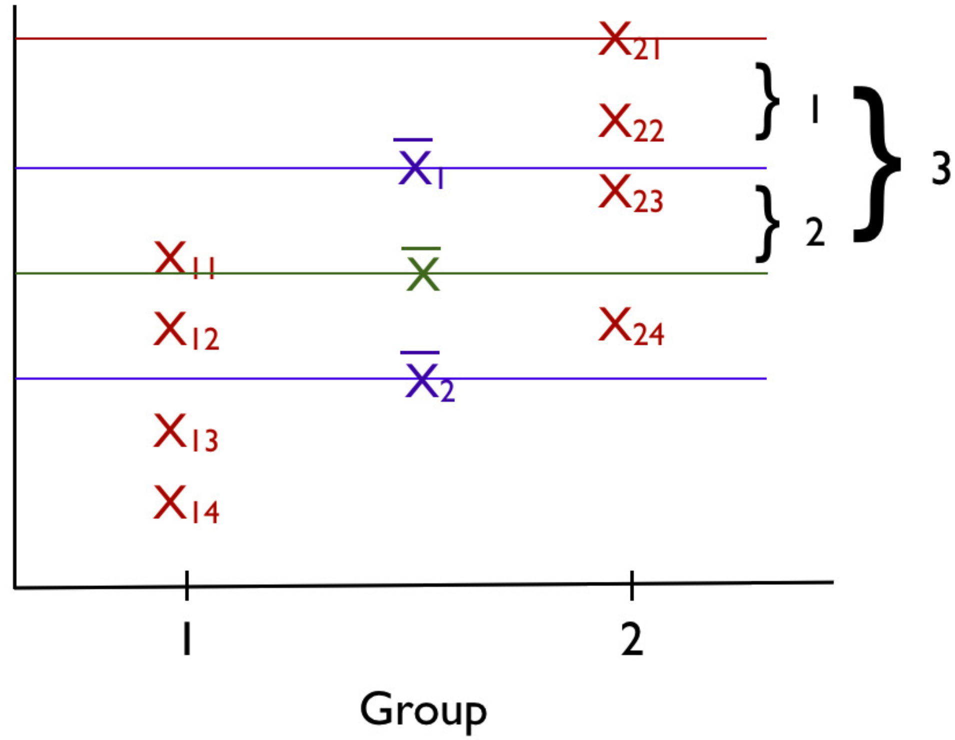
$$\text{WRF} : X_{11}, \ldots, X_{1n_1} \overset{iid}{\sim} N(\mu_1, \sigma^2)$$

$$\text{COAMPS} : X_{21}, \ldots, X_{2n_2} \overset{iid}{\sim} N(\mu_2, \sigma^2)$$

$$\text{Test } H_0 : \mu_1 = \mu_2$$

ANOVA Models

- In a traditional ANOVA model, we decompose deviations from means of batches.



"Total" (3) "Within group" (1) "Between group" (2)

$$(X_{ij} - \bar{X}) = (X_{ij} - \bar{X}_i) + (\bar{X}_i - \bar{X})$$

Compute test statistic based on sums of squares.

OR, EQUIVALENTLY

$$X_{ij} = \bar{X} + (\bar{X}_i - \bar{X}) + (X_{ij} - \bar{X}_i)$$

μ α_i ϵ_{ij}

Estimate the model and test for main effect.

Functional ANOVA models

- To model distributions of functions

$$X_{ij}(s,t) = \mu_i(s,t) + \varepsilon_{ij}(s,t)$$
$$X_{ij}(s,t) = \mu(s,t) + \alpha_i(s,t) + \varepsilon_{ij}(s,t)$$

i models (WRF, COAMPS)
j ensemble members
spatial location s
time t

- Assign spatially correlated Gaussian process (flexible) prior to each, and fit the model and correlations simultaneously

Estimate free parameters instead of physics-based parameters

- No *a priori* mathematical constraints (functional relationships between parameters)
- No physics bounds (e.g. Posselt and Vukicevic 2010, and others)
- Arbitrary degrees of freedom

- Need to specify an appropriate functional form, here guided by ANOVA results

Add model terms (tendencies) q

$$\frac{\partial U}{\partial t} = \ldots + F + q$$

$$q = \sum_k a_k \phi_k$$

We then estimate the coefficients a_k of basis functions f to limit the problem dimensionality to size k. Compare these estimates to the ANOVA error model to determine success.

Tests for successful estimation

- Apply corrections to prior (direct) or in tendencies (e.g. Danforth and Kalnay 2008) in cross-assimilation experiments (can also check longer forecasts)
- Use ANOVA and look for small residuals (i.e. good match between estimated errors and known errors)

Look for causality

- Tendencies from existing physics may help – examine PBL tendency budget for terms that cancel the free tendency (error) terms

Scales and dimensions

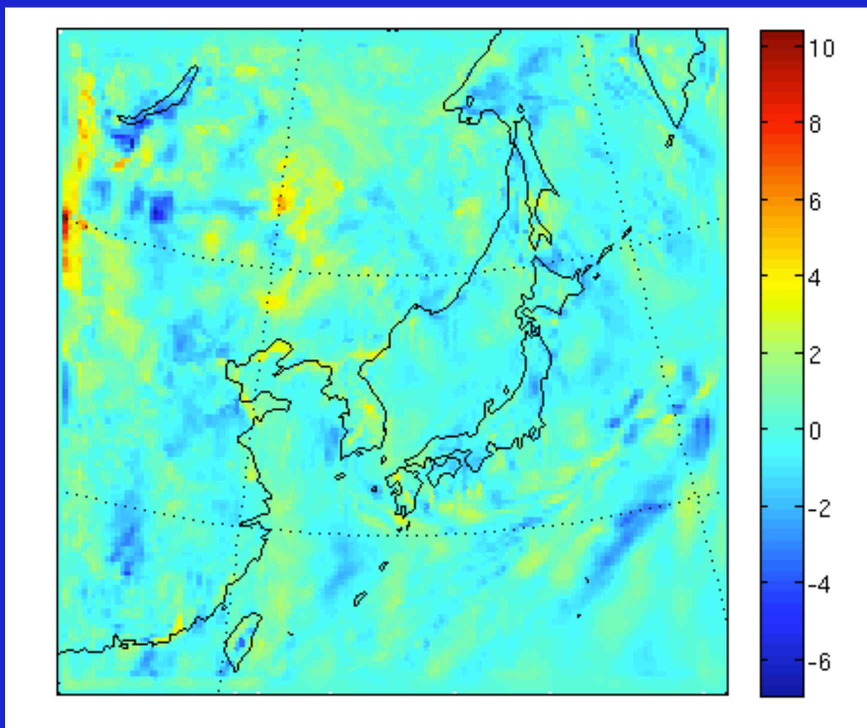
- Preliminary experiment goals:
  - find the number of degrees-of-freedom that we can estimate reliably (in the face of sampling error)
  - find a set of bases for estimation
- Approach:
  - Introduce terms on the RHS (tendencies) that draw from known error structures, guided by ANOVA
  - In otherwise perfect-model experiments, estimate basis coefficients
- Evaluation: compare estimates and imposed errors (again ANOVA) and look for small residuals

Nuts and bolts

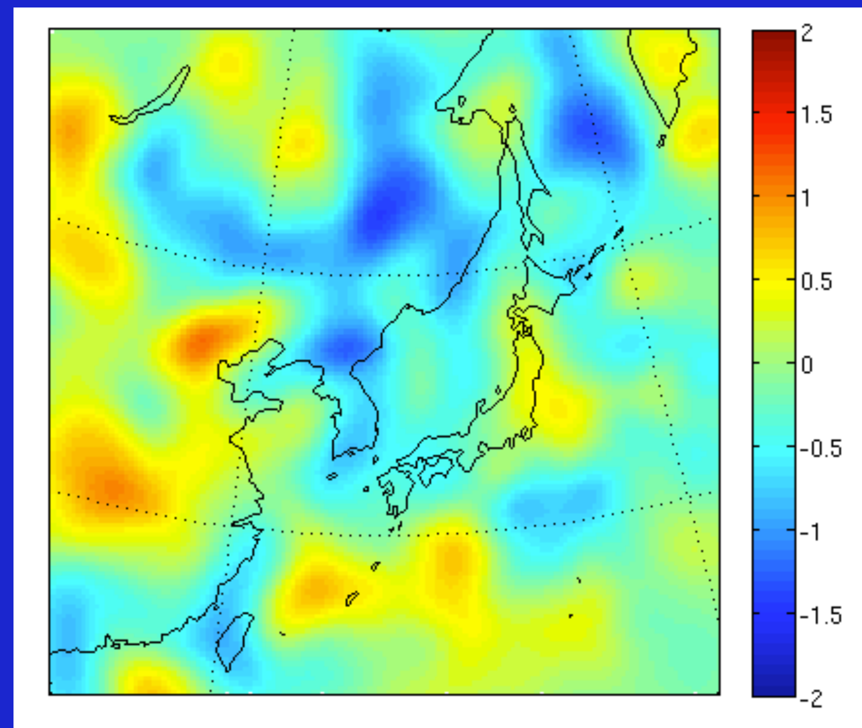
- 96-member NOGAPS ensemble (created using DART) for COAMPS and WRF LBCs
- Start with Dx=30 km
- Domain over E Asia and Sea of Japan
- Stick to PBL winds, use layer-1 winds to start

E.g.: estimating the wrong errors

Prior ensemble-mean error in U (m/s)

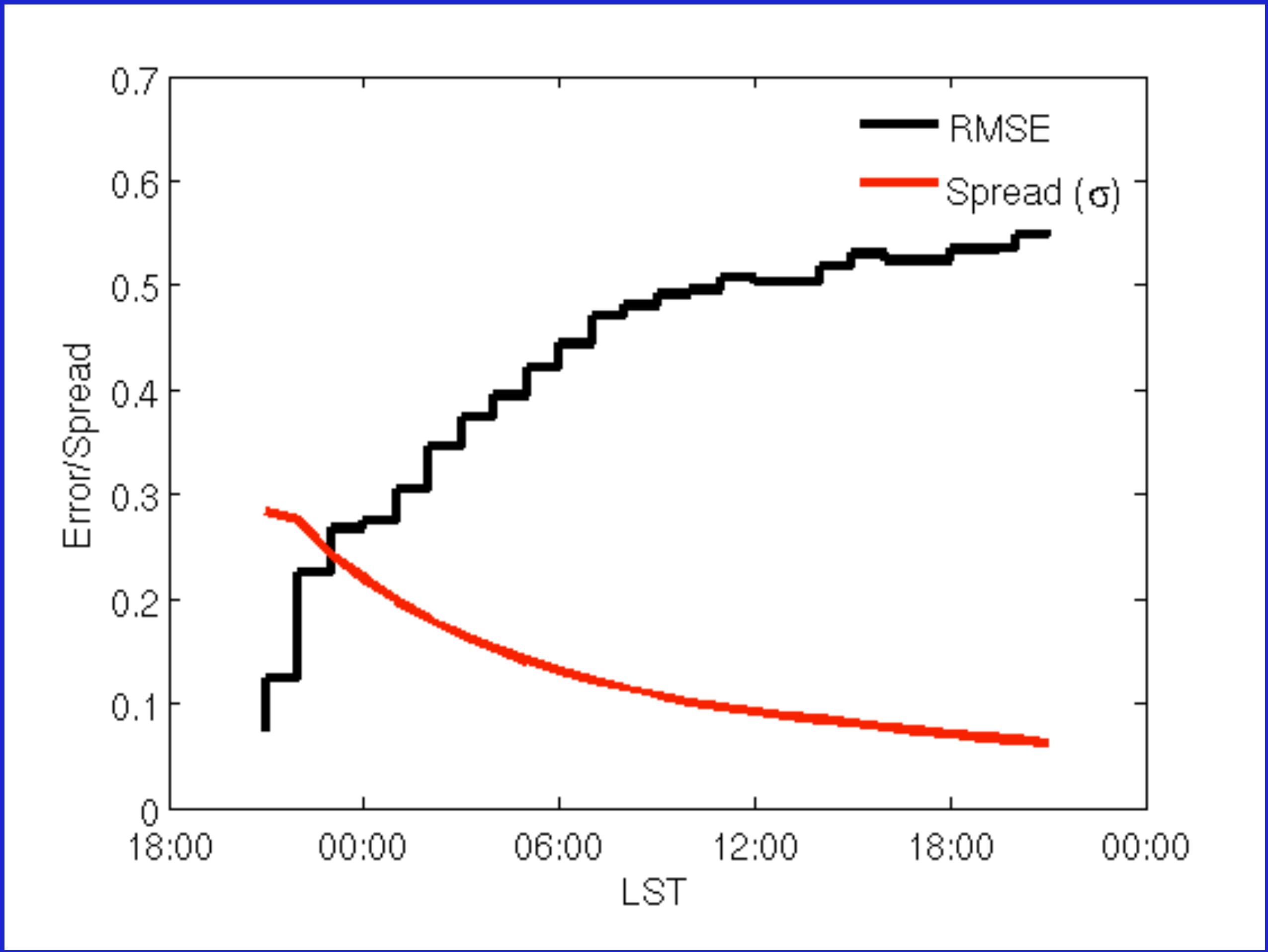


Parameter estimate $q(s)$.



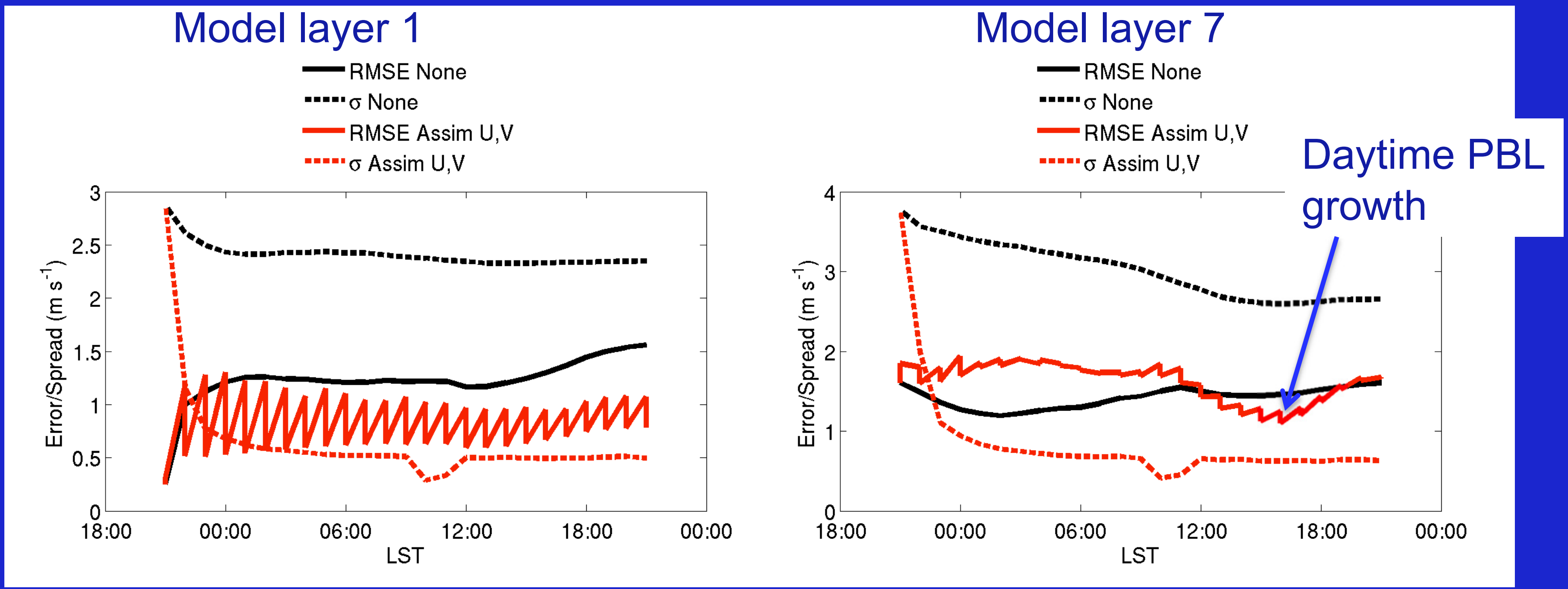
Assimilating model has a U-tendency error of -0.0001 m s^{-2} (known). Here I forgot to apply the tendencies on the RHS, so the parameter estimates do not act on the state. Initial parameter distributions are uniform random and constant in space. These pictures are after 12 h, assimilating hourly.

Parameter estimates



Here RMSE is the RMS (parameter - 0.0001).

Assimilation with deficient model



Black is no assimilation, red is assimilating first-layer U,V hourly. Model is deficient by 0.0001 m s^{-2}. Initial spread (random spatially correlated perturbations) is large, but initial error is only due to sampling error.