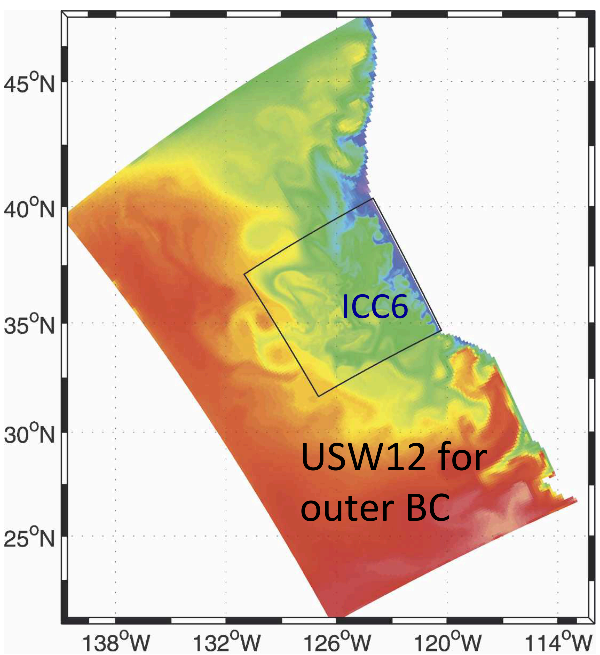
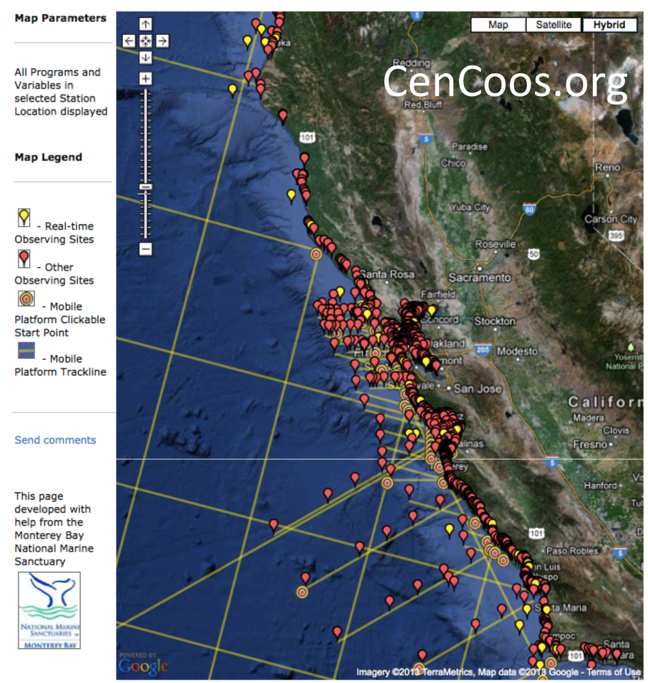
Coping with Multi-Scale & Multi-Resolution in Data Assimilation

Kayo Ide, UMD

Zhijin Li NASA JPL

Jim McWilliams UCLA





Motivation: Dynamics and Model

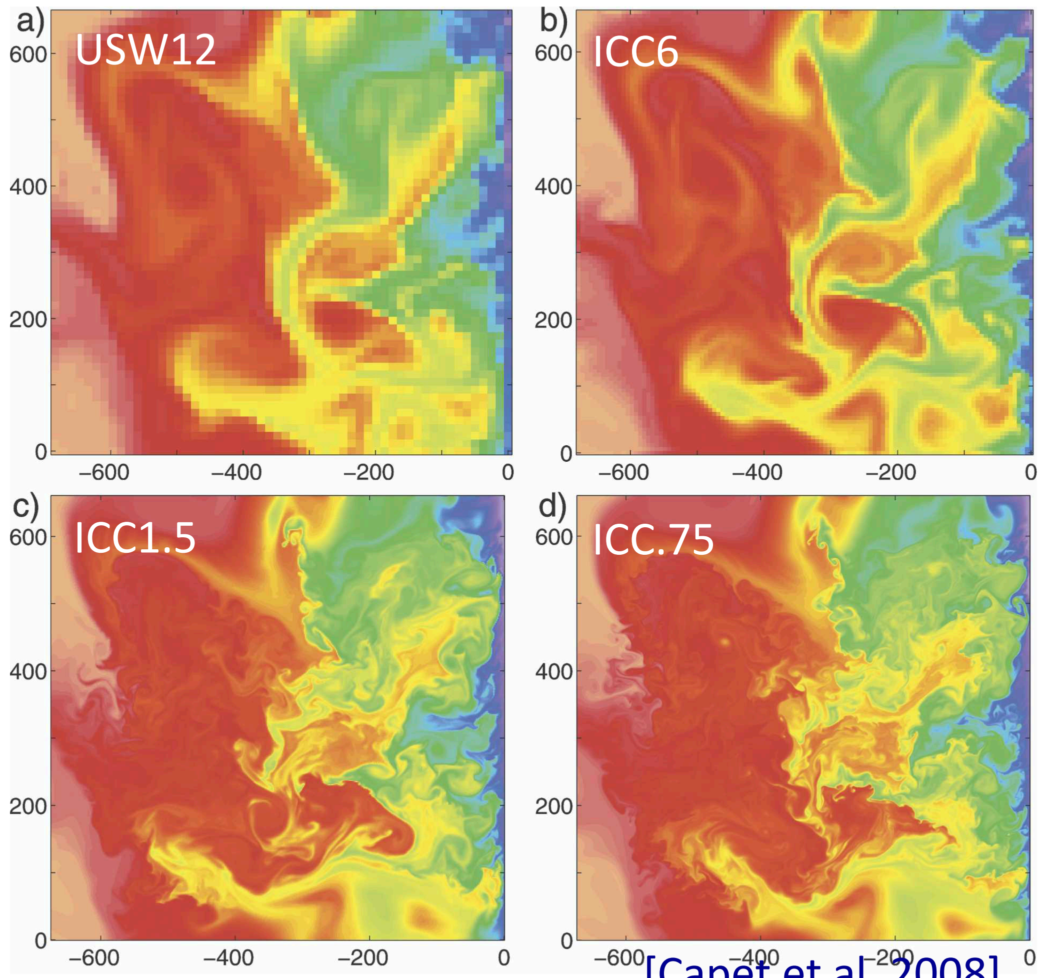
- Scale and Resolution
  - Geophysical dynamics exhibits multi-scale (MS) phenomena
  - Modeling: Multi-Resolution (MR)

- This work focuses on spatially
  - MS/MR in horizontal

$$\mathbf{x} = \mathbf{x}_L + \mathbf{x}_S \quad [+\ldots]$$

$\mathbf{x}_L$: large-scale

$\mathbf{x}_S$: smaller-scale



[Capet et al, 2008]

Motivation: MS Increment

- Single obs illustration
  - Cost function

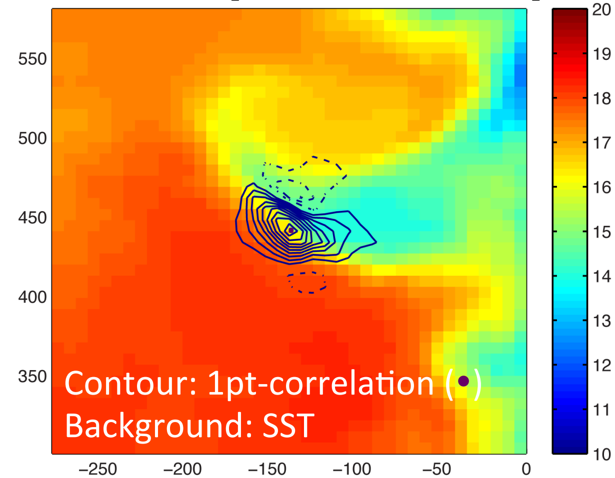
$$J(\Delta\mathbf{x}) = \frac{1}{2}\Delta\mathbf{x}^T(\mathbf{P}^\mathrm{b})^{-1}\Delta\mathbf{x} + \frac{1}{2}(\mathbf{d}-\mathbf{H}\Delta\mathbf{x})^T\mathbf{R}^{-1}(\mathbf{d}-\mathbf{H}\Delta\mathbf{x})$$

  - Increment $\Delta\mathbf{x}^\mathrm{a} = \mathbf{P}^\mathrm{b}\mathbf{H}^\mathrm{T}(\mathbf{H}\mathbf{P}^\mathrm{b}\mathbf{H}^\mathrm{T}+\mathbf{R}^\mathrm{o})^{-1}\,\mathbf{d}$

$\mathbf{d} = \mathbf{y}^\mathrm{o} - \mathbf{H}\mathbf{x}^\mathrm{f}$

$$\Delta\mathbf{x}^\mathrm{a} = \begin{pmatrix} P^\mathrm{b}_{1l} \\ \vdots \\ P^\mathrm{b}_{nl} \\ \vdots \\ P^\mathrm{b}_{Nl} \end{pmatrix} (P^\mathrm{b}_{ll} + R_{ll})^{-1}(y^\mathrm{o}_l - x^\mathrm{b}_l)$$

SST [ICC6 ensemble]



Courtesy of T. Miyoshi

For $\Delta\mathbf{x}^\mathrm{a}$ to be MS, $\mathbf{P}^\mathrm{b}$ must be MS

Motivation: MS Increment

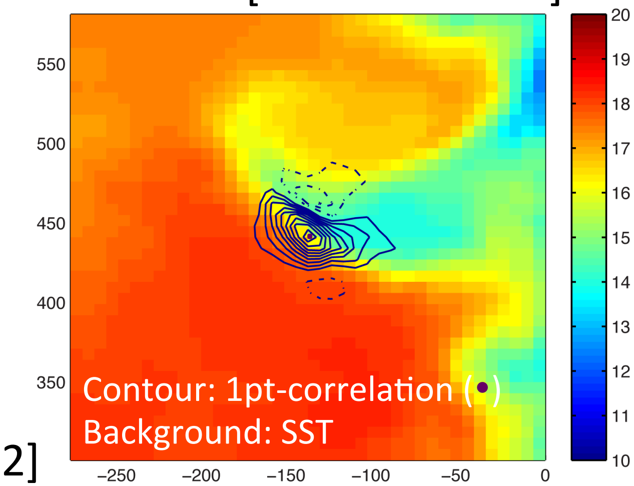
- Standard Var Formulation for MS

1. MS $\mathbf{P}^b = \beta_L \mathbf{P}^b{}_L + \beta_S \mathbf{P}^b{}_S$ (Additive $\mathbf{P}^b$)

$$J(\Delta\mathbf{x}) = \frac{1}{2}\Delta\mathbf{x}^T(\beta_L \mathbf{P}_L^b + \beta_S \mathbf{P}_S^b)^{-1}\Delta\mathbf{x} + \frac{1}{2}(\mathbf{d} - \mathbf{H}\Delta\mathbf{x})^T \mathbf{R}^{-1}(\mathbf{d} - \mathbf{H}\Delta\mathbf{x})$$

[Wu et al, 2002]

SST [ICC6 ensemble]



Courtesy of T. Miyoshi

- Single obs increment

$$\Delta\mathbf{x}^a = \left\{ \beta_L \begin{pmatrix} P^b_{L.1l} \\ \vdots \\ P^b_{L.nl} \\ \vdots \\ P^b_{L.Nl} \end{pmatrix} + \beta_S \begin{pmatrix} P^b_{S.1l} \\ \vdots \\ P^b_{S.nl} \\ \vdots \\ P^b_{S.Nl} \end{pmatrix} \right\} (\beta_L P^b_{L.ll} + \beta_S P^b_{S.ll} + R_{ll})^{-1}(y^o_l - x^b_l)$$

Motivation: MS Increment

- Standard Var Formulations for MS

1. MS $\mathbf{P}^b = \beta_L \mathbf{P}^b{}_L + \beta_S \mathbf{P}^b{}_S$ (Additive $\mathbf{P}^f$)
   (using single control vector)

[Wu et al, 2002]

$$J(\Delta\mathbf{x}) = \frac{1}{2}\Delta\mathbf{x}^T(\beta_L \mathbf{P}_L^b + \beta_S \mathbf{P}_S^b)^{-1}\Delta\mathbf{x} + \frac{1}{2}(\mathbf{d} - \mathbf{H}\Delta\mathbf{x})^T \mathbf{R}^{-1}(\mathbf{d} - \mathbf{H}\Delta\mathbf{x})$$

2. MS $\Delta\mathbf{x}^a = \Delta\mathbf{x}^a{}_L + \Delta\mathbf{x}^a{}_S$ (Additive $\Delta\mathbf{x}^a$)
   (using dual/multi control vectors)

$$J(\Delta\mathbf{x}_L, \Delta\mathbf{x}_S) = \frac{1}{2}(\Delta\mathbf{x}_L)^T(\beta_L \mathbf{P}_L^b)^{-1}\Delta\mathbf{x}_L + \frac{1}{2}(\Delta\mathbf{x}_S)^T(\beta_S \mathbf{P}_S^b)^{-1}\Delta\mathbf{x}_S$$

$$+\frac{1}{2}(\mathbf{d} - \mathbf{H}\Delta\mathbf{x})^T \mathbf{R}^{-1}(\mathbf{d} - \mathbf{H}\Delta\mathbf{x})$$

$$\Delta\mathbf{x} = \Delta\mathbf{x}_L + \Delta\mathbf{x}_S$$

MS-MR Formulation for Background

- MS-MR Variation for $\Delta \mathbf{x}^a = \Delta \mathbf{x}^a{}_L + \Delta \mathbf{x}^a{}_S$

  Scheme 1. Concurrent Decomposition for multi-resolution (MR)

$$J_L(\Delta\mathbf{x}_L) = \frac{1}{2}(\Delta\mathbf{x}_L)^T(\beta_L\mathbf{P}_L^b)^{-1}\Delta\mathbf{x}_L + \frac{1}{2}(\mathbf{H}\Delta\mathbf{x}_L - \mathbf{d})^T(\mathbf{R} + \beta_S\mathbf{H}\mathbf{P}_S^b\mathbf{H}^T)^{-1}(\mathbf{H}\Delta\mathbf{x}_L - \mathbf{d})$$

$$J_S(\Delta\mathbf{x}_S) = \frac{1}{2}(\Delta\mathbf{x}_S)^T(\beta_S\mathbf{P}_S^b)^{-1}\Delta\mathbf{x}_S + \frac{1}{2}(\mathbf{H}\Delta\mathbf{x}_S - \mathbf{d})^T(\mathbf{R} + \beta_L\mathbf{H}\mathbf{P}_L^b\mathbf{H}^T)^{-1}(\mathbf{H}\Delta\mathbf{x}_S - \mathbf{d})$$

Note

» Solutions are the same for $\Delta \mathbf{x}^a = \Delta \mathbf{x}^a{}_L + \Delta \mathbf{x}^a{}_S$ of the concurrent MS decomposition scheme and original MS schemes

» Computationally efficient by

∗ Solving for $\Delta \mathbf{x}^a{}_L$ at low resolution

∗ Solving for $\Delta \mathbf{x}^a{}_S$ with $\beta_S \mathbf{P}^b{}_S$ separately at high resolution

» Easily extended to multi- (more than dual) scales

» $\Delta \mathbf{x}_L$ is usually dominant

$\rightarrow$Solving for $\Delta \mathbf{x}_L$ first gives better initialization for $\Delta \mathbf{x}_S$?

MS-MR Formulation for Background

- MS-MR Variation for $\Delta\mathbf{x}^a = \Delta\mathbf{x}^a{}_L + \Delta\mathbf{x}^a{}_S$

Scheme 2. Successive Approach [Li et al, 2014]

1. Large scale (unchanged)

$$J_L(\Delta\mathbf{x}_L) = \frac{1}{2}(\Delta\mathbf{x}_L)^T(\beta_L\mathbf{P}_L^b)^{-1}\Delta\mathbf{x}_L + \frac{1}{2}(\mathbf{H}\Delta\mathbf{x}_L - \mathbf{d})^T(\mathbf{R} + \beta_S\mathbf{H}\mathbf{P}_S^b\mathbf{H}^T)^{-1}(\mathbf{H}\Delta\mathbf{x}_L - \mathbf{d})$$

2. Small scale (use $\Delta\mathbf{x}^a{}_L$)

$$J_S(\Delta\mathbf{x}_S) = \frac{1}{2}(\Delta\mathbf{x}_S)^T(\beta_S\mathbf{P}_S^b)^{-1}\Delta\mathbf{x}_S + \frac{1}{2}(\mathbf{H}\Delta\mathbf{x}_S - \mathbf{d}_S^*)^T(\mathbf{R}_S^*)^{-1}(\mathbf{H}\Delta\mathbf{x}_S - \mathbf{d}_S^*)$$

where

updated innovation: $\mathbf{d}^*{}_S = \mathbf{d} - \mathbf{H}\Delta\mathbf{x}^a{}_L$

updated obs error covariance: $\mathbf{R}^*{}_S = \mathbf{R} + \beta_L\mathbf{H}\mathbf{P}^a{}_L\mathbf{H}^T$

Note

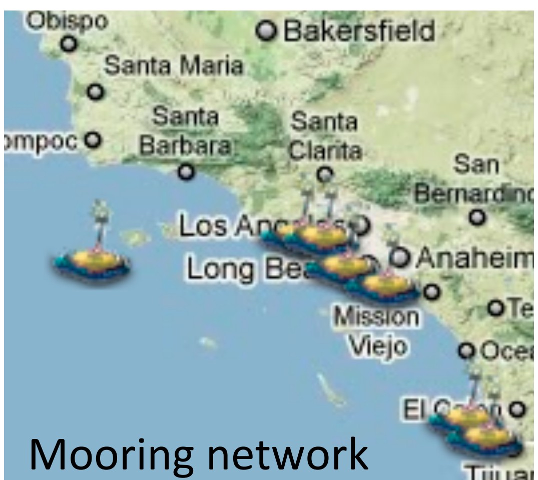
» Similar to Successive Covariance Localization (SCL: Zhang et al, 2009)

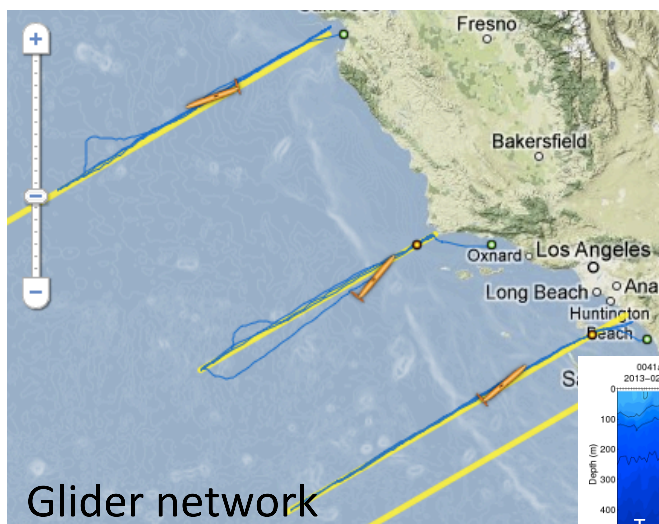
Motivation: MR in Observing System

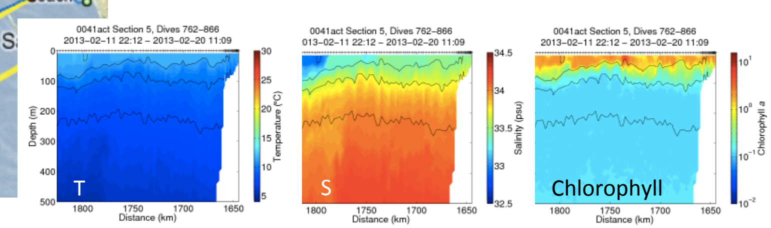
- Spatial density of observing system varies from one obs type to another
  - Coarse (inhomogeneous):

$$\mathbf{y}_C = \mathbf{H}_C\mathbf{x} = \mathbf{H}_C(\mathbf{x}_L + \mathbf{x}_S) \qquad \text{with } \mathbf{R}_C$$

    - Mooring
    - Gliders







Motivation: MR in Observing System

- Spatial density of observing system varies from one obs type to another
  - Coarse network:

$$\mathbf{y}_C = \mathbf{H}_C\mathbf{x} = \mathbf{H}_C(\mathbf{x}_L + \mathbf{x}_S) \qquad \text{with } \mathbf{R}_C$$

    – Mooring

    – Gliders

  - Dense network

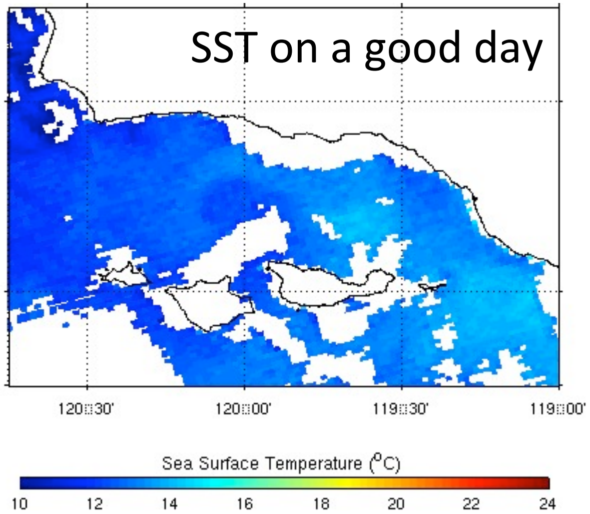
$$\mathbf{y}_D = \mathbf{H}_D\mathbf{x} = \mathbf{H}_D(\mathbf{x}_L + \mathbf{x}_S)$$

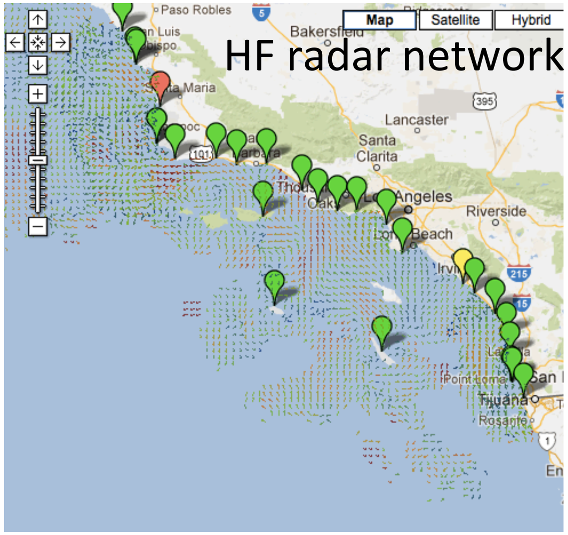
$$= \mathbf{y}_{D.L} + \mathbf{y}_{D.S}$$

$$\begin{pmatrix} \mathbf{y}_{D,L} \\ \mathbf{y}_{D,S} \end{pmatrix} = \begin{pmatrix} \mathbf{H}_D\mathbf{x}_L \\ \mathbf{H}_D\mathbf{x}_S \end{pmatrix} \quad \text{with} \begin{pmatrix} \mathbf{R}_{D.L} \\ \mathbf{R}_{D.S} \end{pmatrix}$$

    – Satellite images (SST) at surface

    – HR radar (surface velocity)





MS-MR Formulation for Observation

- MS-MR Variation for $\Delta\mathbf{x}^a = \Delta\mathbf{x}^a{}_L + \Delta\mathbf{x}^a{}_S$
  - Observation

$$\mathbf{y} = \begin{pmatrix} \mathbf{y}_C \\ \mathbf{y}_{D.L} \\ \mathbf{y}_{D.S} \end{pmatrix} = \begin{pmatrix} \mathbf{H}_C(\mathbf{x}_L + \mathbf{x}_S) \\ \mathbf{H}_D\mathbf{x}_L \\ \mathbf{H}_D\mathbf{x}_S \end{pmatrix} \text{ with } \begin{pmatrix} \mathbf{R}_C \\ \mathbf{R}_{D.L} \\ \mathbf{R}_{D.S} \end{pmatrix}$$

← Obs that cannot be decomposed

← Obs that can be decomposed

  - Scheme 1. MS VAR by scale-dependent decomposition

$$J_L(\Delta\mathbf{x}_L) = \frac{1}{2}(\Delta\mathbf{x}_L)^T(\beta_L\mathbf{P}_L^b)^{-1}\Delta\mathbf{x}_L + \frac{1}{2}(\mathbf{d}_C - \mathbf{H}_C\Delta\mathbf{x}_L)^T(\mathbf{R}_C + \beta_S\mathbf{H}_C\mathbf{P}_S^b\mathbf{H}_C{}^T)^{-1}(\mathbf{d}_C - \mathbf{H}_C\Delta\mathbf{x}_L)$$
$$+\frac{1}{2}(\mathbf{d}_{D.L} - \mathbf{H}_D\Delta\mathbf{x}_L)^T(\mathbf{R}_{D.L})^{-1}(\mathbf{d}_{D.L} - \mathbf{H}_D\Delta\mathbf{x}_L)$$

$$J_S(\Delta\mathbf{x}_S) = \frac{1}{2}(\Delta\mathbf{x}_S)^T(\beta_S\mathbf{P}_S^b)^{-1}\Delta\mathbf{x}_S + \frac{1}{2}(\mathbf{d}_C - \mathbf{H}_C\Delta\mathbf{x}_S)^T(\mathbf{R}_C + \beta_L\mathbf{H}_C\mathbf{P}_L^b\mathbf{H}_C{}^T)^{-1}(\mathbf{d}_C - \mathbf{H}_C\Delta\mathbf{x}_S)$$
$$+\frac{1}{2}(\mathbf{d}_C - \mathbf{H}_C\Delta\mathbf{x}_S)^T(\mathbf{R}_{D.S})^{-1}(\mathbf{d}_C - \mathbf{H}_C\Delta\mathbf{x}_S)$$

MS-MR VAR

- Scheme 2. Successive Implementation
  - Large-Scale:

$$J_L(\Delta\mathbf{x}_L)=\frac{1}{2}(\Delta\mathbf{x}_L)^T(\beta_L\mathbf{P}_L^b)^{-1}\Delta\mathbf{x}_L+\frac{1}{2}(\mathbf{d}_C-\mathbf{H}_C\Delta\mathbf{x}_L)^T(\mathbf{R}_C+\beta_S\mathbf{H}_C\mathbf{P}_S^b\mathbf{H}_C{}^T)^{-1}(\mathbf{d}_C-\mathbf{H}_C\Delta\mathbf{x}_L)$$

$$+\frac{1}{2}(\mathbf{d}_{D.L}-\mathbf{H}_D\Delta\mathbf{x}_L)^T(\mathbf{R}_{D.L})^{-1}(\mathbf{d}_{D.L}-\mathbf{H}_D\Delta\mathbf{x}_L)$$

  - Small-scale:

$$J_S(\Delta\mathbf{x}_S)=\frac{1}{2}(\Delta\mathbf{x}_S)^T(\beta_S\mathbf{P}_S^b)^{-1}\Delta\mathbf{x}_S+\frac{1}{2}(\mathbf{d}^*_{C.S}-\mathbf{H}_C\Delta\mathbf{x}_S)^T(\mathbf{R}^*_{C.S}{}^T)^{-1}(\mathbf{d}^*_{C.S}-\mathbf{H}_C\Delta\mathbf{x}_S)$$

$$+\frac{1}{2}(\mathbf{d}^*_{D.S}-\mathbf{H}_D\Delta\mathbf{x}_S)^T(\mathbf{R}^*_{D.S})^{-1}(\mathbf{d}^*_{D.S}-\mathbf{H}_D\Delta\mathbf{x}_S)$$

Successive Correction by large scale analysis

$$\mathbf{d}^*_{D/C.S}=\mathbf{d}_{D/C}-\mathbf{H}_{D/C}\Delta\mathbf{x}_L{}^a \quad \text{with} \quad \mathbf{R}^*_{D.S/C}=\mathbf{R}_{D/C.S}+\beta_L\mathbf{H}_{D/C}\mathbf{P}_L^a\mathbf{H}_{D/C}{}^T$$

» Estimation of dynamically important Large-scale first, then higher density to better capture smaller scales in the successive assimilation

VAR to (L)ETKF

- VAR

$$J(\Delta\mathbf{x}) = \frac{1}{2}\Delta\mathbf{x}^T(\mathbf{P}^b)^{-1}\Delta\mathbf{x} + \frac{1}{2}(\mathbf{d}-\mathbf{H}\Delta\mathbf{x})^T\mathbf{R}^{-1}(\mathbf{d}-\mathbf{H}\Delta\mathbf{x})$$

where $\Delta\mathbf{x}^a$ is obtained by optimization

Technically

– Analysis Increment

$$\Delta\mathbf{x}^a = \mathbf{P}^a\mathbf{H}^T\mathbf{R}^{-1}\mathbf{d}$$

– Analysis covariance

$$\mathbf{P}^a = \{(\mathbf{P}^b)^{-1} + \mathbf{H}^T\mathbf{R}^{-1}\mathbf{H}\}^{-1}$$

- (L)ETKF

$$J(\mathbf{w}) = \frac{M-1}{2}\mathbf{w}^T\mathbf{w} + \frac{1}{2}(\mathbf{d}-\hat{\mathbf{Y}}\mathbf{w})^T\mathbf{R}^{-1}(\mathbf{d}-\hat{\mathbf{Y}}\mathbf{w}); \quad \hat{\mathbf{Y}} = \mathbf{H}\hat{\mathbf{X}}$$

where $\Delta\mathbf{x}^a = \hat{\mathbf{X}}^b\mathbf{w}^a$ using ensemble

– weights $\mathbf{w}$

– spread $\hat{\mathbf{X}}^b \Rightarrow \mathbf{P}^b = (M-1)^{-1}\hat{\mathbf{X}}^b(\hat{\mathbf{X}}^b)^T$

Solutions are

– Mean $\Delta\mathbf{x}^a = \hat{\mathbf{X}}^b\mathbf{w}^a$

$$\mathbf{w}^a = \hat{\mathbf{P}}^a\mathbf{H}^T\mathbf{R}^{-1}\mathbf{d}$$

– Spread $\hat{\mathbf{X}}^a = \hat{\mathbf{X}}^b\mathbf{W}^a$

$$\hat{\mathbf{P}}^a = \{(M-1)\mathbf{I} + \hat{\mathbf{Y}}^T\mathbf{R}^{-1}\hat{\mathbf{Y}}\}^{-1} = (\hat{\mathbf{W}}^a)^2$$

MS-MR LETKF Cost Function

- Ensemble representation needed in LETKF

Analysis increment $\Delta\mathbf{x}^{a} = \hat{\mathbf{X}}_{L}^{b}\mathbf{w}_{L}^{a} + \hat{\mathbf{X}}_{S}^{b}\mathbf{w}_{S}^{a}$

Analysis Perturbation $\hat{\mathbf{X}}^{a} = (\beta_{L})^{1/2}\hat{\mathbf{X}}_{L}^{b}\mathbf{W}_{L}^{a} + (\beta_{S})^{1/2}\hat{\mathbf{X}}_{S}^{b}\mathbf{W}_{S}^{a}$

- Cost functions
  - Large-scale

$$J_{L}(\mathbf{w}_{L}) = \frac{1}{2}\frac{M-1}{\beta_{L}}(\mathbf{w}_{L})^{T}\mathbf{w}_{L} + \frac{1}{2}(\mathbf{d}_{C} - \hat{\mathbf{Y}}_{C.L}\mathbf{w}_{L})^{T}\rho_{C.L}\circ(\mathbf{R}_{C} + \frac{\beta_{S}}{M-1}\hat{\mathbf{Y}}_{C.S}\hat{\mathbf{Y}}_{C.S}{}^{T})^{-1}(\mathbf{d}_{C} - \hat{\mathbf{Y}}_{C.L}\mathbf{w}_{L})$$
$$+\frac{1}{2}(\mathbf{d}_{D.L} - \hat{\mathbf{Y}}_{D.L}\mathbf{w}_{L})^{T}\rho_{D.L}\circ(\mathbf{R}_{D.L})^{-1}(\mathbf{d}_{D.L} - \hat{\mathbf{Y}}_{D.L}\mathbf{w}_{L})$$

  - Small-scale:

$$J_{S}(\mathbf{w}_{S}) = \frac{1}{2}\frac{M-1}{\beta_{S}}(\mathbf{w}_{S})^{T}\mathbf{w}_{S} + \frac{1}{2}(\mathbf{d}_{C} - \hat{\mathbf{Y}}_{C.S}\mathbf{w}_{S})^{T}\rho_{C.S}\circ(\mathbf{R}_{C} + \frac{\beta_{L}}{M-1}\hat{\mathbf{Y}}_{C.L}\hat{\mathbf{Y}}_{C.L}{}^{T})^{-1}(\mathbf{d}_{C} - \hat{\mathbf{Y}}_{C.S}\mathbf{w}_{S})$$
$$+\frac{1}{2}(\mathbf{d}_{D.S} - \hat{\mathbf{Y}}_{D.S}\mathbf{w}_{S})^{T}\rho_{D.S}\circ(\mathbf{R}_{D.S})^{-1}(\mathbf{d}_{D.S} - \hat{\mathbf{Y}}_{D.S}\mathbf{w}_{S})$$

MS-MR LETKF

- Concurrent scheme:

  Analysis increment $\Delta \mathbf{x}^a = \hat{\mathbf{X}}_L^b \mathbf{w}_L^a + \hat{\mathbf{X}}_S^b \mathbf{w}_S^a$

  Analysis Perturbation $\hat{\mathbf{X}}^a = (\beta_L)^{1/2} \hat{\mathbf{X}}_L^b \mathbf{W}_L^a + (\beta_S)^{1/2} \hat{\mathbf{X}}_S^b \mathbf{W}_S^a$

  - Large-Scale:

$$\mathbf{w}_L^a = \hat{\mathbf{P}}_L^a \ [\ \hat{\mathbf{Y}}_{C.L}{}^T\ \rho_{C.L} \circ \{\mathbf{R}_C + \frac{\beta_S}{M-1} \hat{\mathbf{Y}}_{C.S} \hat{\mathbf{Y}}_{C.S}{}^T\}^{-1} \mathbf{d}_C + \hat{\mathbf{Y}}_{D.L}{}^T\ \rho_{D.L} \circ (\mathbf{R}_{D.L})^{-1} \mathbf{d}_{D.L}\ ]$$

$$(\hat{\mathbf{W}}_L^a)^{-2} = (\hat{\mathbf{P}}_L^a)^{-1} = \frac{M-1}{\beta_L} \mathbf{I} + \hat{\mathbf{Y}}_{C.L}\ \rho_{C.L} \circ \{\mathbf{R}_C + \frac{\beta_S}{M-1} \hat{\mathbf{Y}}_{C.S} \hat{\mathbf{Y}}_{C.S}{}^T\}^{-1} \hat{\mathbf{Y}}_{C.L}{}^T + \hat{\mathbf{Y}}_{D.L}\ \rho_{D.L} \circ (\mathbf{R}_{D.L})^{-1} \hat{\mathbf{Y}}_{D.L}{}^T$$

  - Small-scale:

$$\mathbf{w}_S^a = \hat{\mathbf{P}}_S^a \ [\ \hat{\mathbf{Y}}_S{}^T\ \rho_{C.S} \circ \{\mathbf{R}_C + \frac{\beta_L}{M-1} \hat{\mathbf{Y}}_{C.L} \hat{\mathbf{Y}}_{C.L}{}^T\}^{-1} \mathbf{d}_C + \hat{\mathbf{Y}}_{D.S}{}^T\ \rho_{D.S} \circ (\mathbf{R}_{D.S})^{-1} \mathbf{d}_{D.S}]$$

$$(\hat{\mathbf{W}}_S^a)^{-2} = (\hat{\mathbf{P}}_S^a)^{-1} = \frac{M-1}{\beta_L} \mathbf{I} + \hat{\mathbf{Y}}_{C.S}\ (\rho_{C.S} \circ \mathbf{R}_C + \frac{\beta_L}{M-1} \hat{\mathbf{Y}}_{C.L} \hat{\mathbf{Y}}_{C.L}{}^T)^{-1} \hat{\mathbf{Y}}_{C.S}{}^T + \hat{\mathbf{Y}}_{D.S}\ \rho_{D.S} \circ (\mathbf{R}_{D.S})^{-1}\ \hat{\mathbf{Y}}_{D.S}{}^T$$

MS-MR :LTKF Cost Function

- Ensemble representation needed in LETKF

Analysis increment $\Delta \mathbf{x}^a = \hat{\mathbf{X}}_L^b \mathbf{w}_L^a + \hat{\mathbf{X}}_S^b \mathbf{w}_S^a$

Analysis Perturbation $\hat{\mathbf{X}}^a = (\beta_L)^{1/2} \hat{\mathbf{X}}_L^b \mathbf{W}_L^a + (\beta_S)^{1/2} \hat{\mathbf{X}}_S^b \mathbf{W}_S^a$

MS-MR multiplicative inflation

- Cost functions
  - Large-scale $\mathbf{H}_C \Delta \mathbf{x}_L = \hat{\mathbf{Y}}_{C.L} \mathbf{w}_L$ & $\mathbf{H}_D \Delta \mathbf{x}_L = \hat{\mathbf{Y}}_{D.L} \mathbf{w}_L$

$$J_L(\mathbf{w}_L) = \frac{1}{2}\frac{M-1}{\beta_L}(\mathbf{w}_L)^T \mathbf{w}_L + \frac{1}{2}(\mathbf{d}_C - \hat{\mathbf{Y}}_{C.L}\mathbf{w}_L)^T \rho_{C.L} \circ (\mathbf{R}_C + \frac{\beta_S}{M-1}\hat{\mathbf{Y}}_{C.S}\hat{\mathbf{Y}}_{C.S}{}^T)^{-1}(\mathbf{d}_C - \hat{\mathbf{Y}}_{C.L}\mathbf{w}_L)$$
$$+ \frac{1}{2}(\mathbf{d}_{D.L} - \hat{\mathbf{Y}}_{D.L}\mathbf{w}_L)^T \rho_{D.L} \circ (\mathbf{R}_{D.L})^{-1}(\mathbf{d}_{D.L} - \hat{\mathbf{Y}}_{D.L}\mathbf{w}_L)$$

  - Small-scale: $\mathbf{H}_C \Delta \mathbf{x}_L = \hat{\mathbf{Y}}_{C.L} \mathbf{w}_L$ & $\mathbf{H}_D \Delta \mathbf{x}_L = \hat{\mathbf{Y}}_{D.L} \mathbf{w}_L$

$$J_S(\mathbf{w}_S) = \frac{1}{2}\frac{M-1}{\beta_S}(\mathbf{w}_S)^T \mathbf{w}_S + \frac{1}{2}(\mathbf{d}_C - \hat{\mathbf{Y}}_{C.S}\mathbf{w}_S)^T \rho_{C.S} \circ (\mathbf{R}_C + \frac{\beta_L}{M-1}\hat{\mathbf{Y}}_{C.L}\hat{\mathbf{Y}}_{C.L}{}^T)^{-1}(\mathbf{d}_C - \hat{\mathbf{Y}}_{C.S}\mathbf{w}_S)$$
$$+ \frac{1}{2}(\mathbf{d}_{D.S} - \hat{\mathbf{Y}}_{D.S}\mathbf{w}_S)^T \rho_{D.S} \circ (\mathbf{R}_{D.S})^{-1}(\mathbf{d}_{D.S} - \hat{\mathbf{Y}}_{D.S}\mathbf{w}_S)$$

MS-MR localization

MS-MR (L)ETKF

- Successive scheme:

Analysis increment $\Delta\mathbf{x}^a = \hat{\mathbf{X}}_L^b \mathbf{w}_L^a + \hat{\mathbf{X}}_S^b \mathbf{w}_S^a$

Analysis Perturbation $\hat{\mathbf{X}}^a = (\beta_L)^{1/2} \hat{\mathbf{X}}_L^b \mathbf{W}_L^a + (\beta_S)^{1/2} \hat{\mathbf{X}}_S^b \mathbf{W}_S^a$

- Large-Scale:

$$\mathbf{w}_L^a = \hat{\mathbf{P}}_L^a \left[\hat{\mathbf{Y}}_{C.L}{}^T \rho_{C.L} \circ \{\mathbf{R}_C + \frac{\beta_S}{M-1} \hat{\mathbf{Y}}_{C.S} \hat{\mathbf{Y}}_{C.S}{}^T\}^{-1} \mathbf{d}_C + \hat{\mathbf{Y}}_{D.L}{}^T \rho_{D.L} \circ (\mathbf{R}_{D.L})^{-1} \mathbf{d}_{D.L} \right]$$

$$(\hat{\mathbf{W}}_L^a)^{-2} = (\hat{\mathbf{P}}_L^a)^{-1} = \frac{M-1}{\beta_L}\mathbf{I} + \hat{\mathbf{Y}}_{C.L} \rho_{C.L} \circ \{\mathbf{R}_C + \frac{\beta_S}{M-1} \hat{\mathbf{Y}}_{C.S} \hat{\mathbf{Y}}_{C.S}{}^T\}^{-1} \hat{\mathbf{Y}}_{C.L}{}^T + \hat{\mathbf{Y}}_{D.L} \rho_{D.L} \circ (\mathbf{R}_{D.L})^{-1} \hat{\mathbf{Y}}_{D.L}{}^T$$

- Small-scale:

$$\mathbf{w}_S^a = \hat{\mathbf{P}}_S^a \left[\hat{\mathbf{Y}}_S^{*T} \rho_{C.S} \circ \{\mathbf{R}_C + \frac{\beta_L}{M-1} \hat{\mathbf{Y}}_{C.L}^* \hat{\mathbf{Y}}_{C.L}^*{}^T\}^{-1} \mathbf{d}_C^* + \hat{\mathbf{Y}}_{D.S}^*{}^T \rho_{D.S} \circ (\mathbf{R}_{D.S})^{-1} \mathbf{d}_{D.S}^* \right]$$

$$(\hat{\mathbf{W}}_S^a)^{-2} = (\hat{\mathbf{P}}_S^a)^{-1} = \frac{M-1}{\beta_L}\mathbf{I} + \hat{\mathbf{Y}}_{C.S}^* (\rho_{C.S} \circ \mathbf{R}_C + \frac{\beta_L}{M-1} \hat{\mathbf{Y}}_{C.L}^* \hat{\mathbf{Y}}_{C.L}^*{}^T)^{-1} \hat{\mathbf{Y}}_{C.S}^*{}^T + \hat{\mathbf{Y}}_{D.S}^* \rho_{D.S} \circ (\mathbf{R}_{D.S})^{-1} \hat{\mathbf{Y}}_{D.S}^*{}^T$$

Mid-Point Remarks

- Issue
  - Scale separation, particularly $\mathbf{y}_D$ & $\mathbf{R}_D$

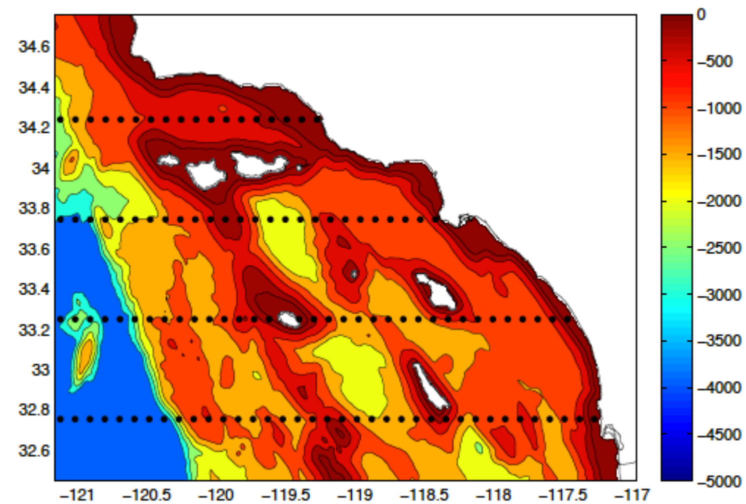
- Applications
  - MS3DVAR: developed for Southern California Bight (SCB)
  - MS-MR LETKF: being implemented into SCB & Lorenz MS model
  - Hybrid: developed & to be implemented in the SCB & Chesapeake Bay

- Examples
  - SCB
    - OSSE
    - Real data experiments
  - Illustration using simple 1D example

California Coastal Ocean Data Assimilation System

- Observing System Simulation Experiments (OSSEs) Setup
  - Model: Regional Ocean Modeling System (ROMS)
    - Resolution: 1km x 40 levels nested in low-resolution model
    - Atmos forcing: WRF at 2km
  - Southern California Coastal Ocean Observing System (SCCOS)
    - SST at 2km resolution
    - Surface (u,v) at 2km resolution
    - T/S profiling along 4 tracks at
      - » 60km<D_L separation between trac
      - » 10km<D_s, D_{3Dvar} along track

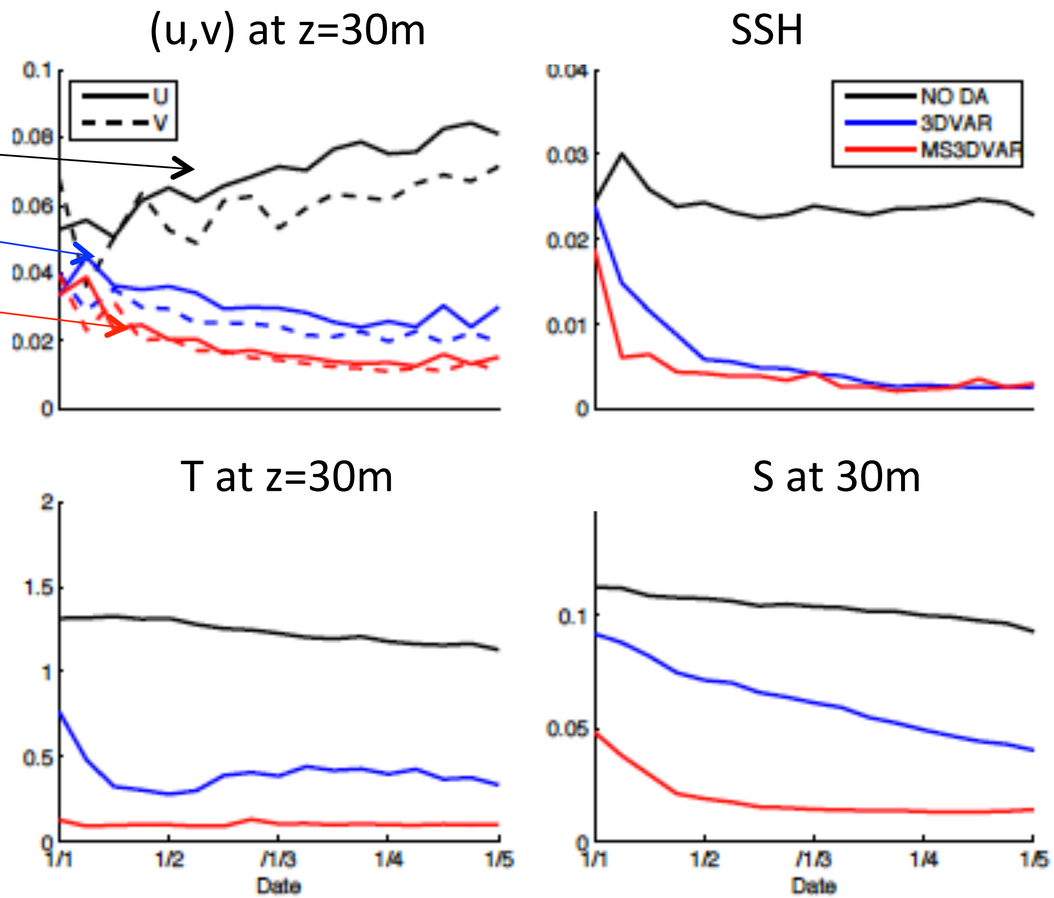
      Up to 400m
    - *Balance (geostrophic & hydrostatic) is incorporated*



Bathymetry and
OSSE T/S profiling positions

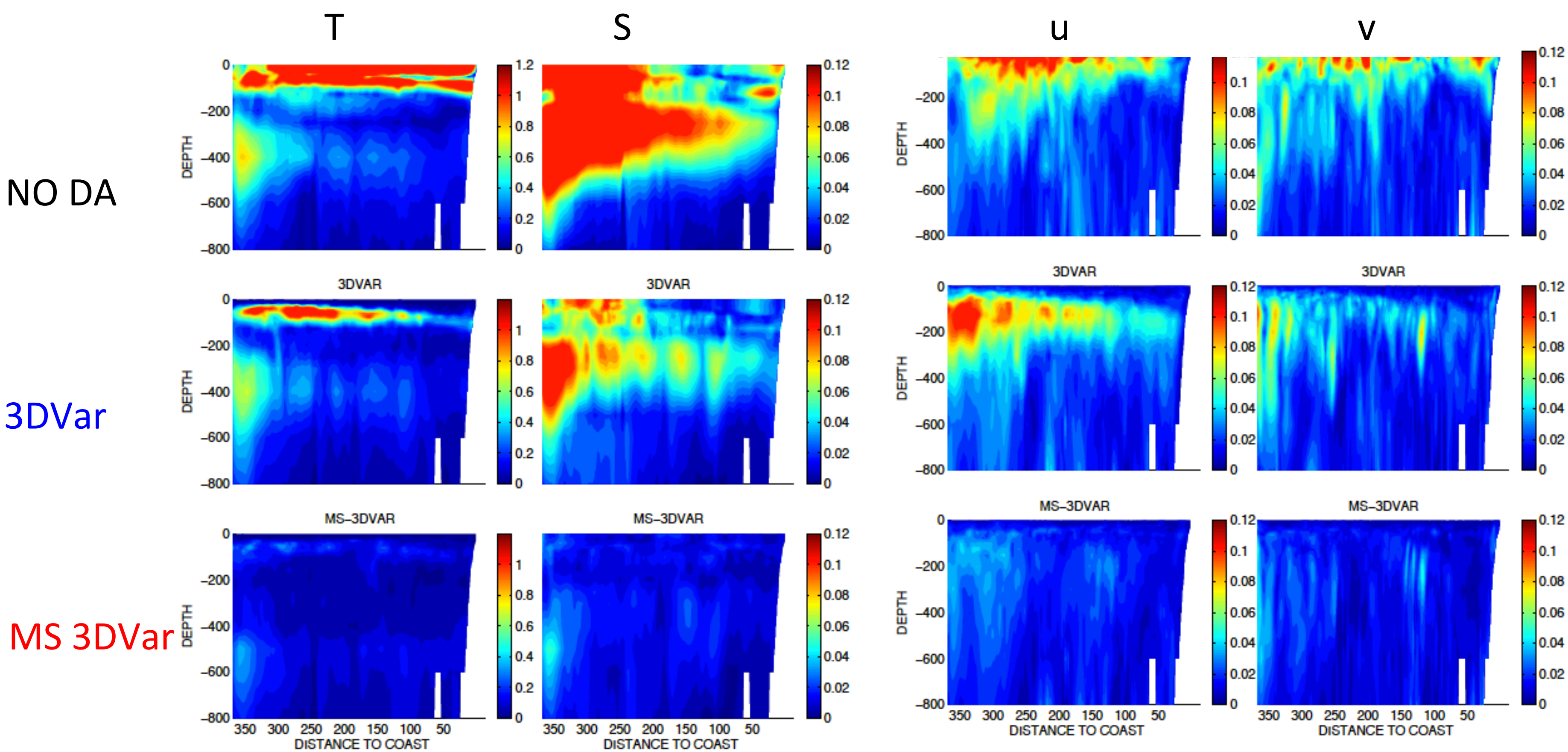
OSSE: RMSE Analysis Error in Time

- Comparison between
  - NO DA
  - Standard 3DVar (D_s in **B**)
  - MS 3DVar (**B** & H)
    - Spin-up faster and converges to smaller RMSE



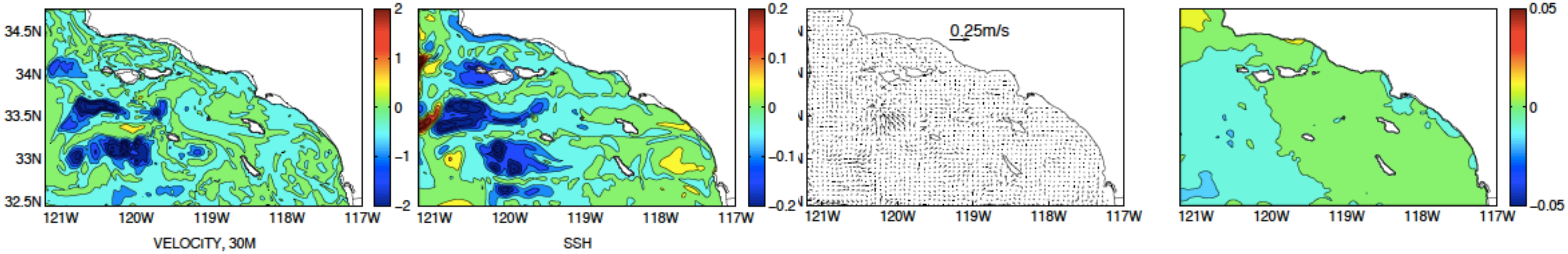
OSSE: RMSE Analysis Error

- Vertical distribution of analysis RMSE At Day 3 (along-shore average)

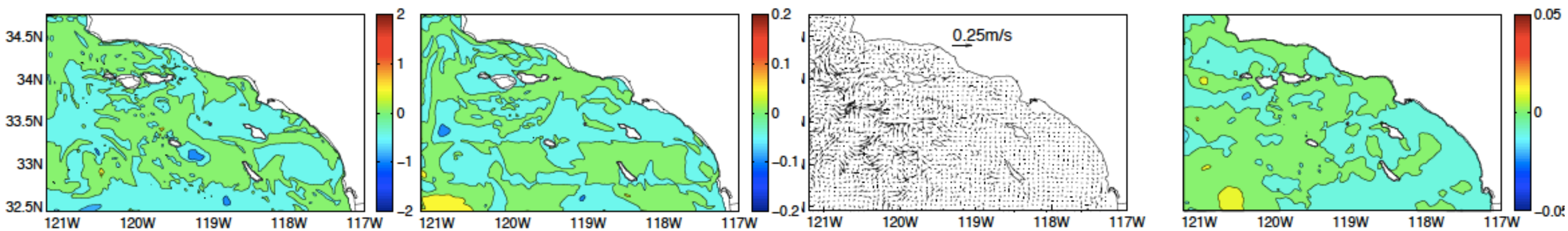


OSSE: Instantaneous Error. z=30m, day 4

- Standard 3DVar



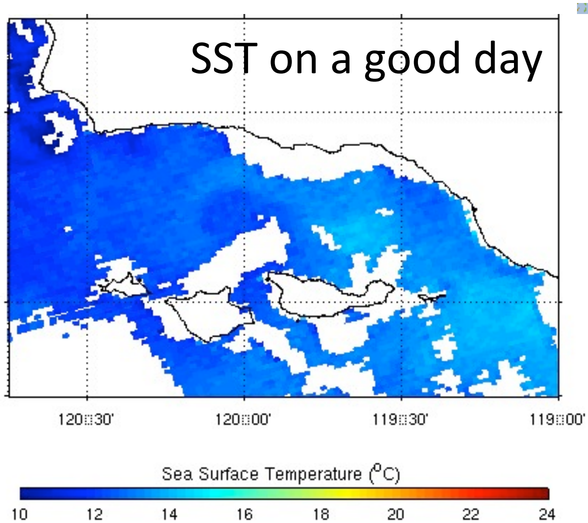
- MS 3DVar



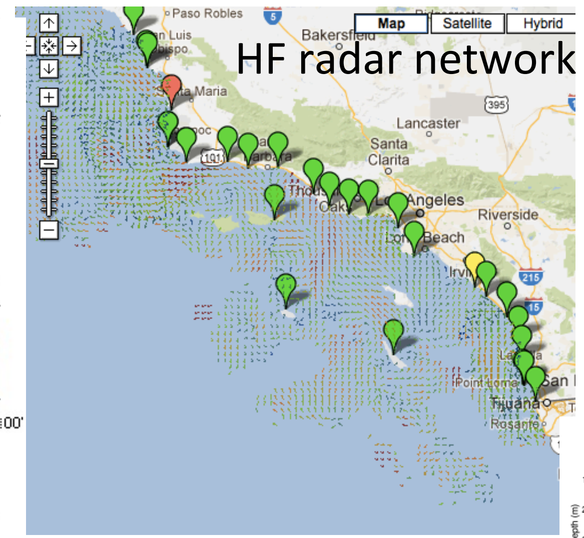
T at z=30m S at z=30m (u,v) at 30m SSH

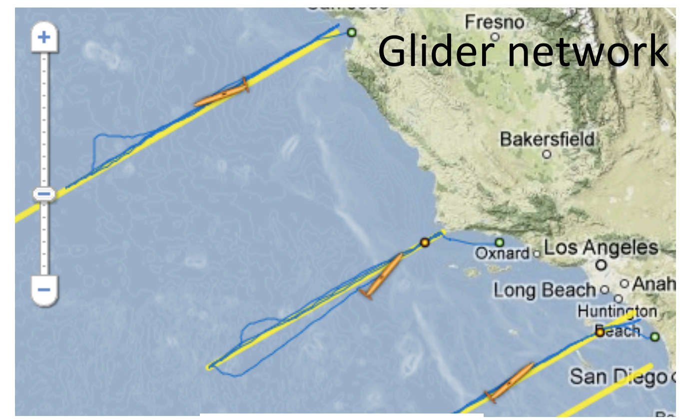
California Coastal Ocean Data Assimilation System

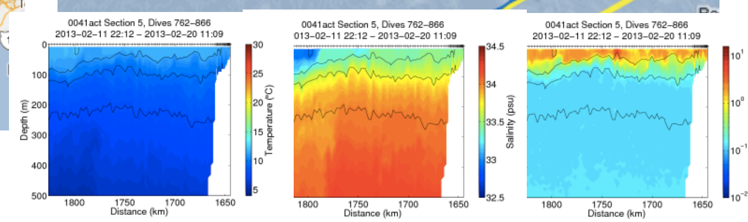
- Real Observation Experiments
  - Initialization: 01/01/2008
  - Observing system (H)



http://sccoos.org



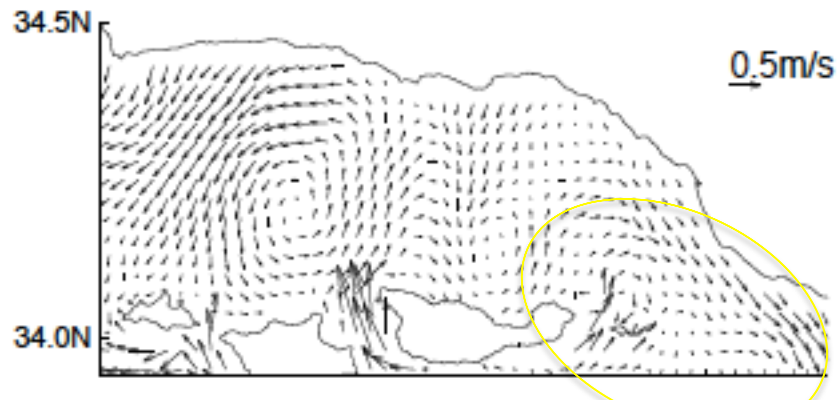




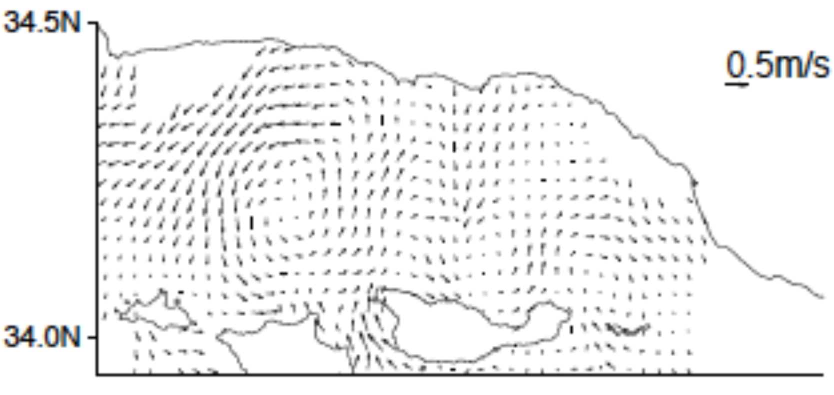
- Performance: Comparison against independent data for bias
  - – No DA
  - – Standard 3DVar
  - – MS 3DVar

Real Observation Experiments

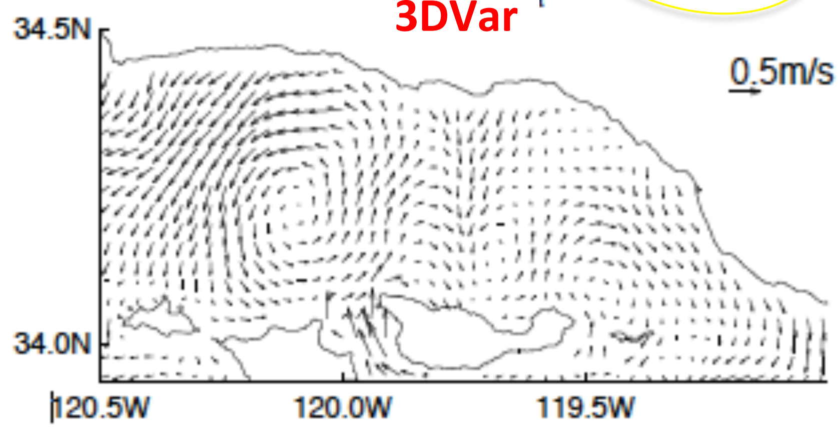
- HF Radar: 03UTC, August 13, 2008: difference in details
  - 3DVar
    - Corr (u,v)=(0.53,0.67)
    - RMSD (u,v)=(0.19,0.15) [m/s]
  - MS 3DVar
    - Corr (u,v)=(0.61,0.72)
    - RMSD (u,v)=(0.17,0.14) [m/s]



3DVar



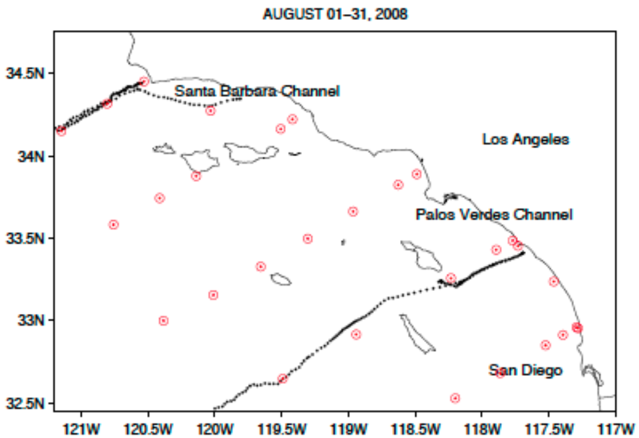
HF Radar (Obs)

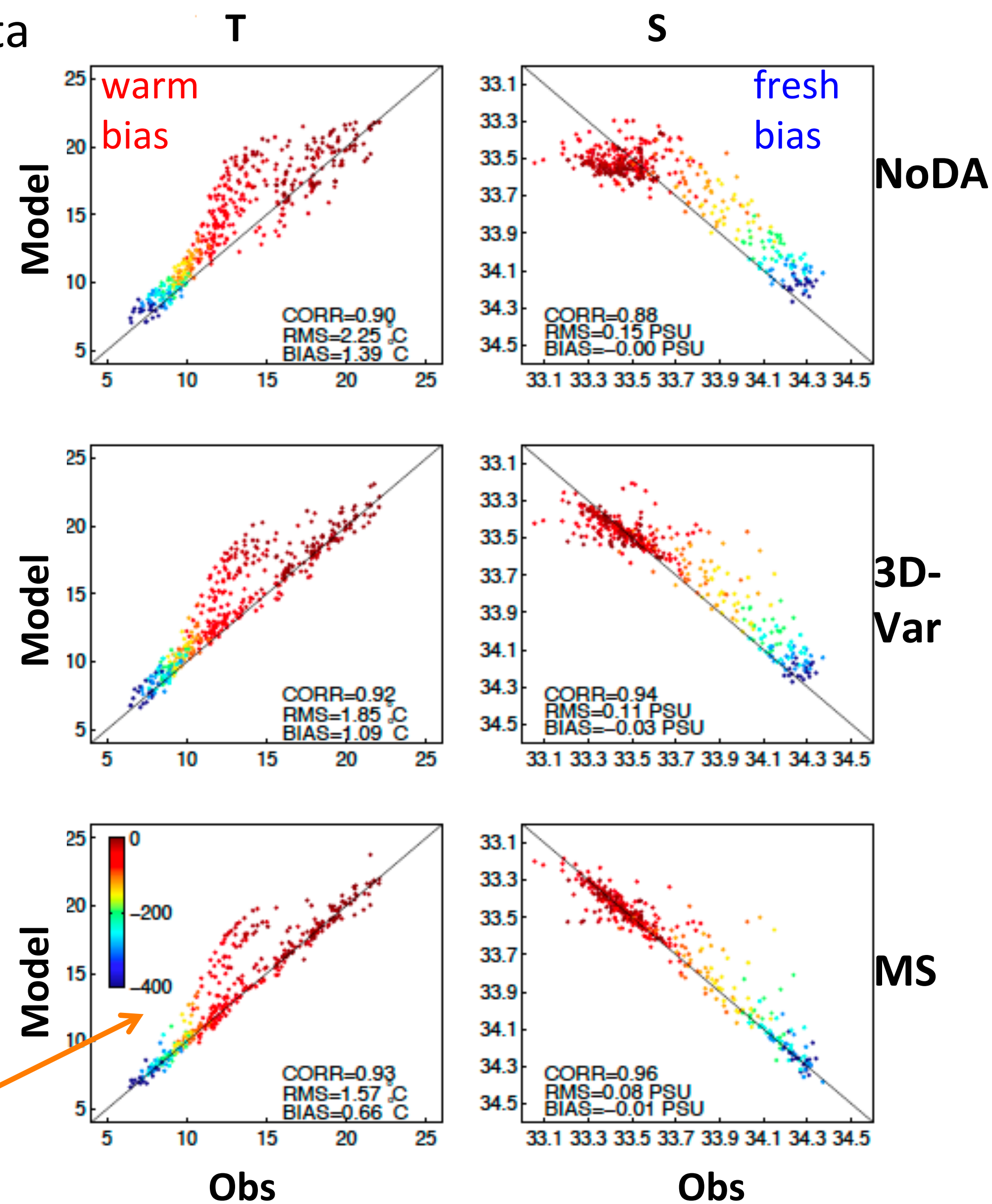


MS 3DVar

California Coastal Ocean Data Assimilation System

- Comparison with independent data [08/14-29/2008]
  - CalCOFI location (o)





- MS 3DVar
  - Reduction of error
  - Reduction of bias
- In general
  - More observations needed

Simple Demonstration (1D-Var)

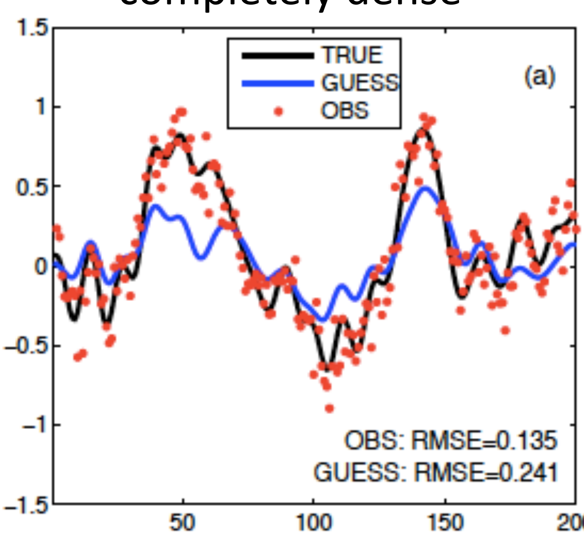
- Experimental setup for MS/AB with $\{D_L, D_S\}$ & SS with $\{D_L, D_{m1}, D_{m2}, D_S\}$
  - **x**:
    – $\mathbf{x}^t$ is MS $\quad x_n^t = S_0 \sum_{k=1}^{K} a_k^t \cos(\frac{k\pi n}{N} + \phi_n^t)$: $\quad a_k^t = k^{-\gamma}$ with $\gamma \subset [0,2]$

    – $\mathbf{x}^b$ is MS $\quad x_n^b = S_0 \sum_{k=1}^{K} a_k^b \cos(\frac{k\pi n}{N} + \phi_n^b)$: $\quad a_k^b = p_0 \lambda_k a_k^t$ with $\lambda_k \subset U(0,1)$

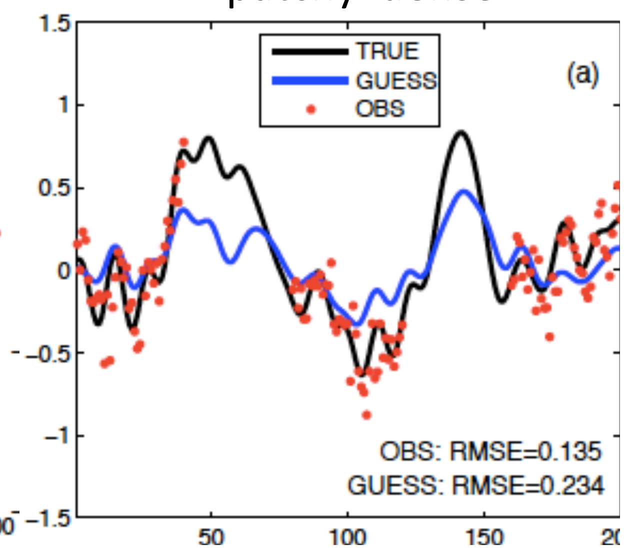
    – $\mathbf{P}^b$ may be MS/AB with $(D_L, D_S)=(40, 5)$ & properly estimated (σ^b_L, σ^b_S)

    may be SS with D =40, 20, 10, 5 & properly estimated σ^b
  - **y=Hx** may be MR

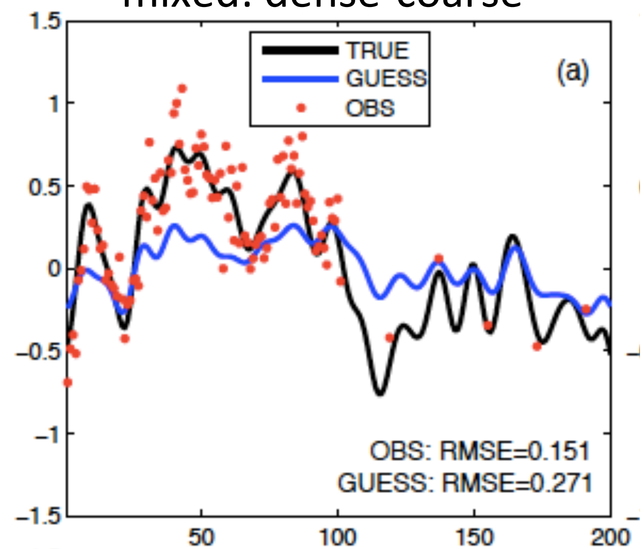
completely dense



patchy dense

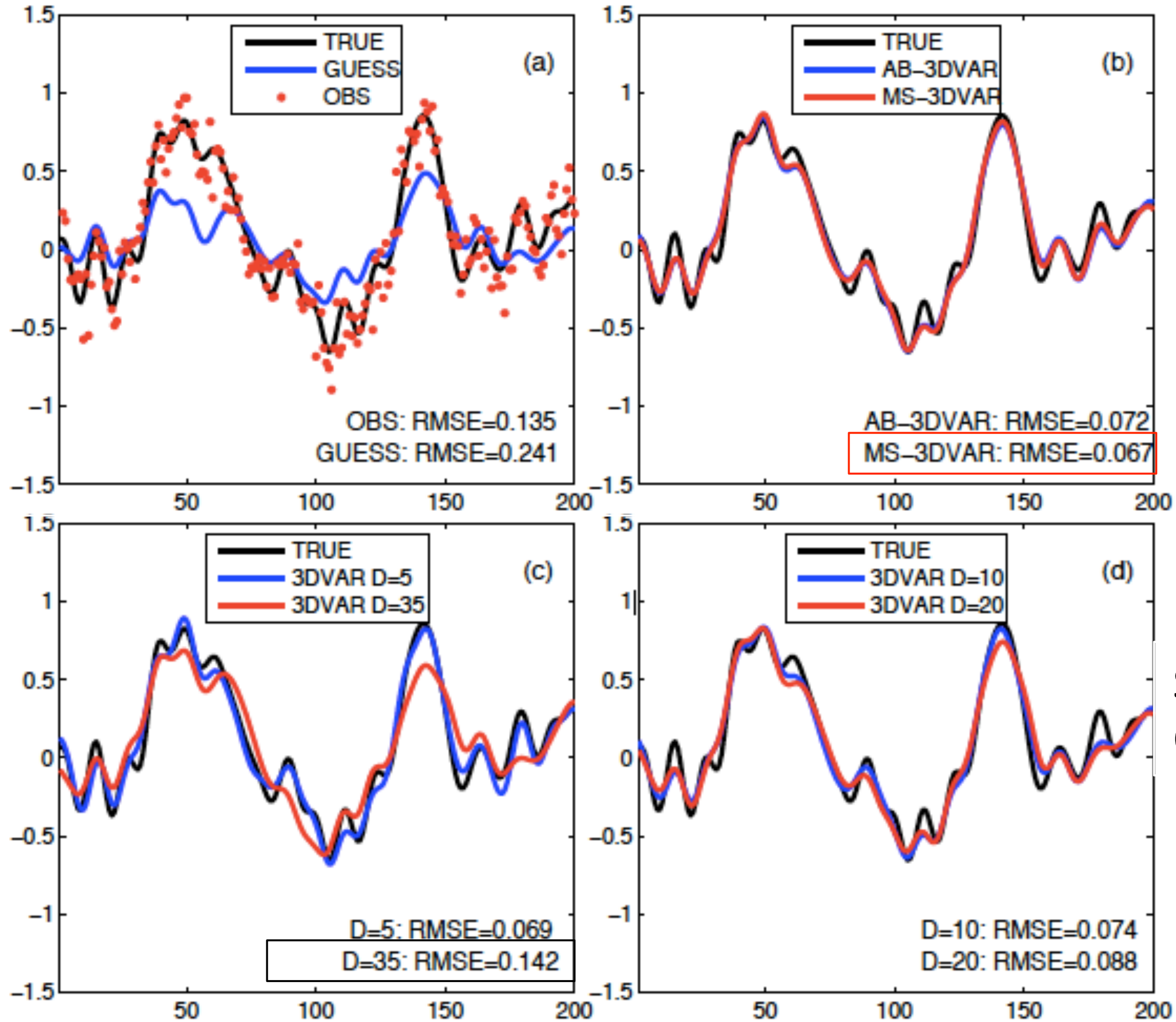


mixed: dense-coarse



Completely Dense Observing Network

- Analysis ($\mathbf{x}^a$)



AB and MS work we with D_S & D_L

SS (Single Scale)

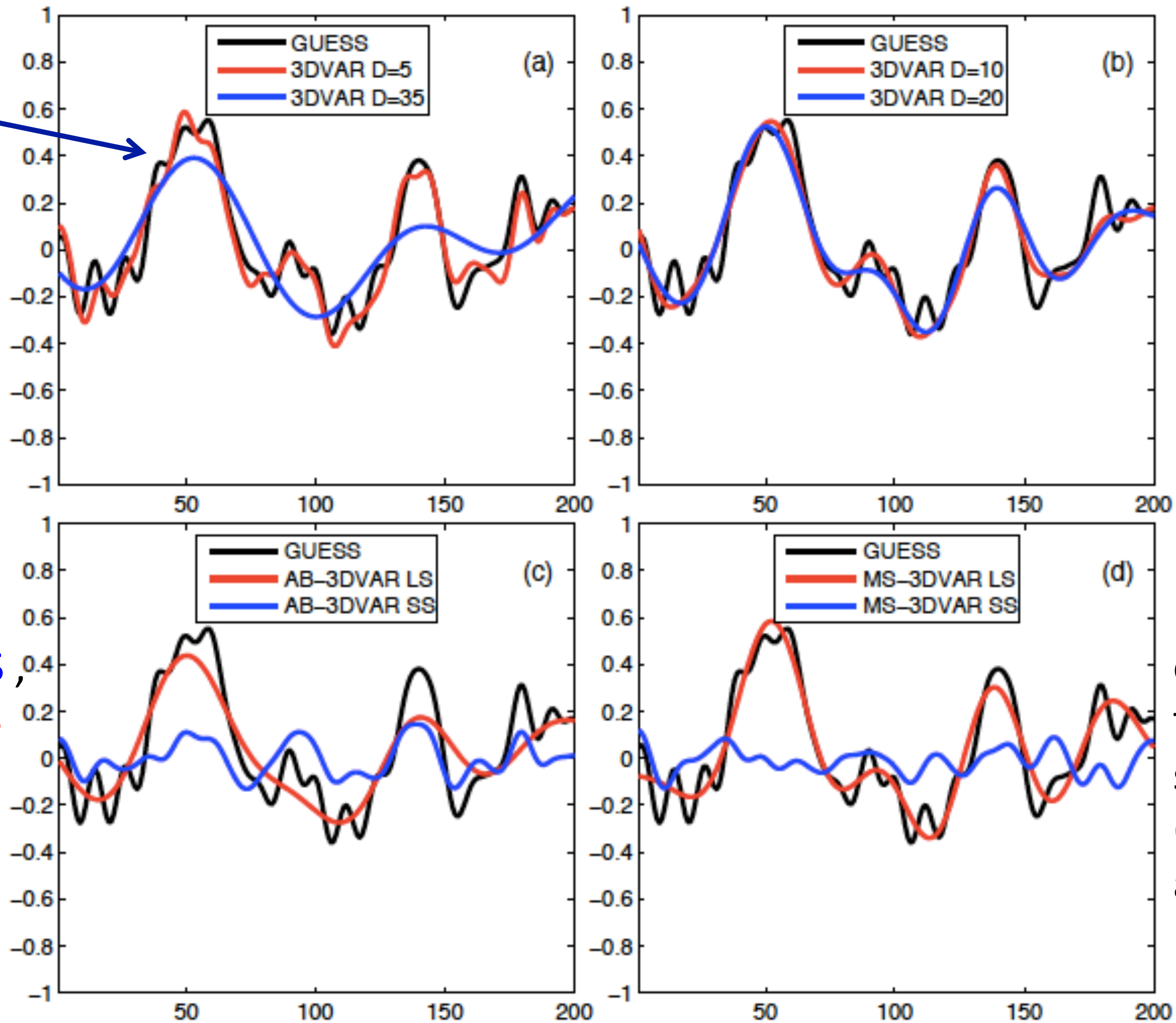
3DVar works OK at D_S not at D_L

SS 3DVar works OK at med. D

Completely Dense Observing Network

- Analysis Increment ($\Delta \mathbf{x}^a$)

Black: target ($\mathbf{x}^t$-$\mathbf{x}^b$)



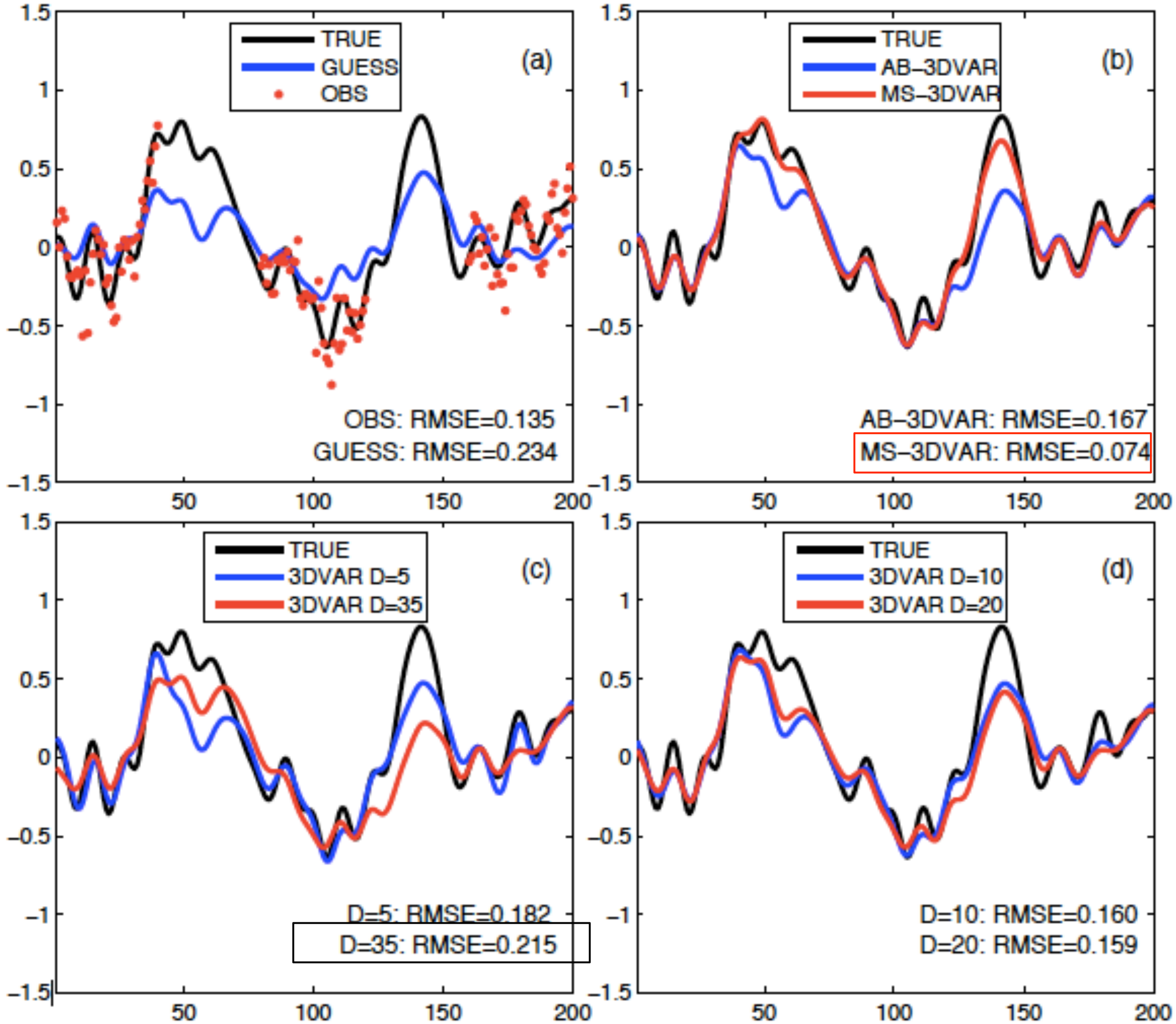
3DVar works better at med. D than D_S or D_L

AB and MS work Differently at LS , and hence at SS

MS captures LS more effectively than AB by sequential (successive) approach

Patchy-Dense Observation Network

- Analysis ($\mathbf{x}^a$)



MS works much better than AB

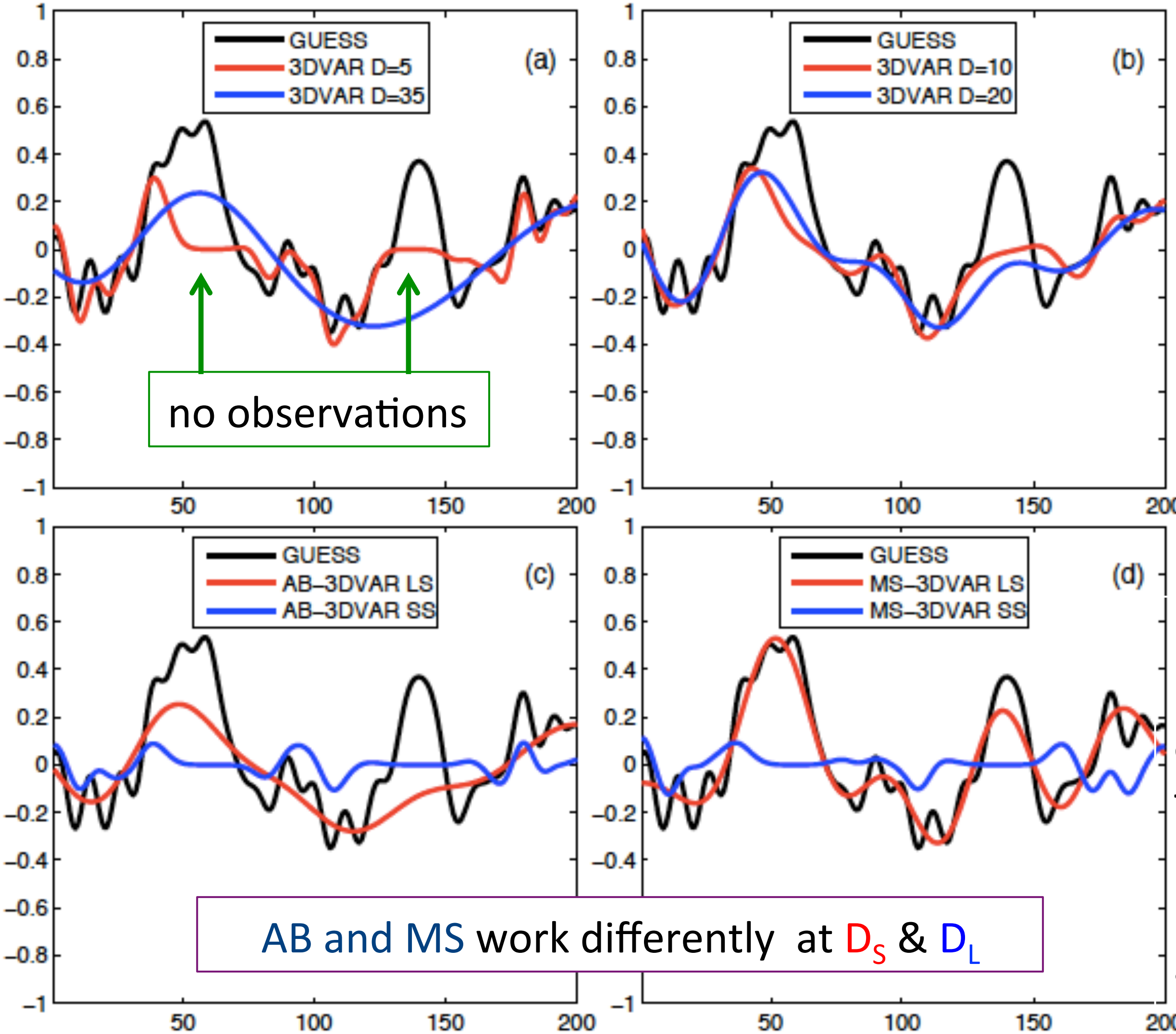
3DVAR don't work well: Better at D_S than D_L

3DVAR works OK at med. D

Patchy-Dense Observation Network

- Analysis Increment ($\Delta\mathbf{x}^a$)

3DVar don't work well: Better at D_S than D_L

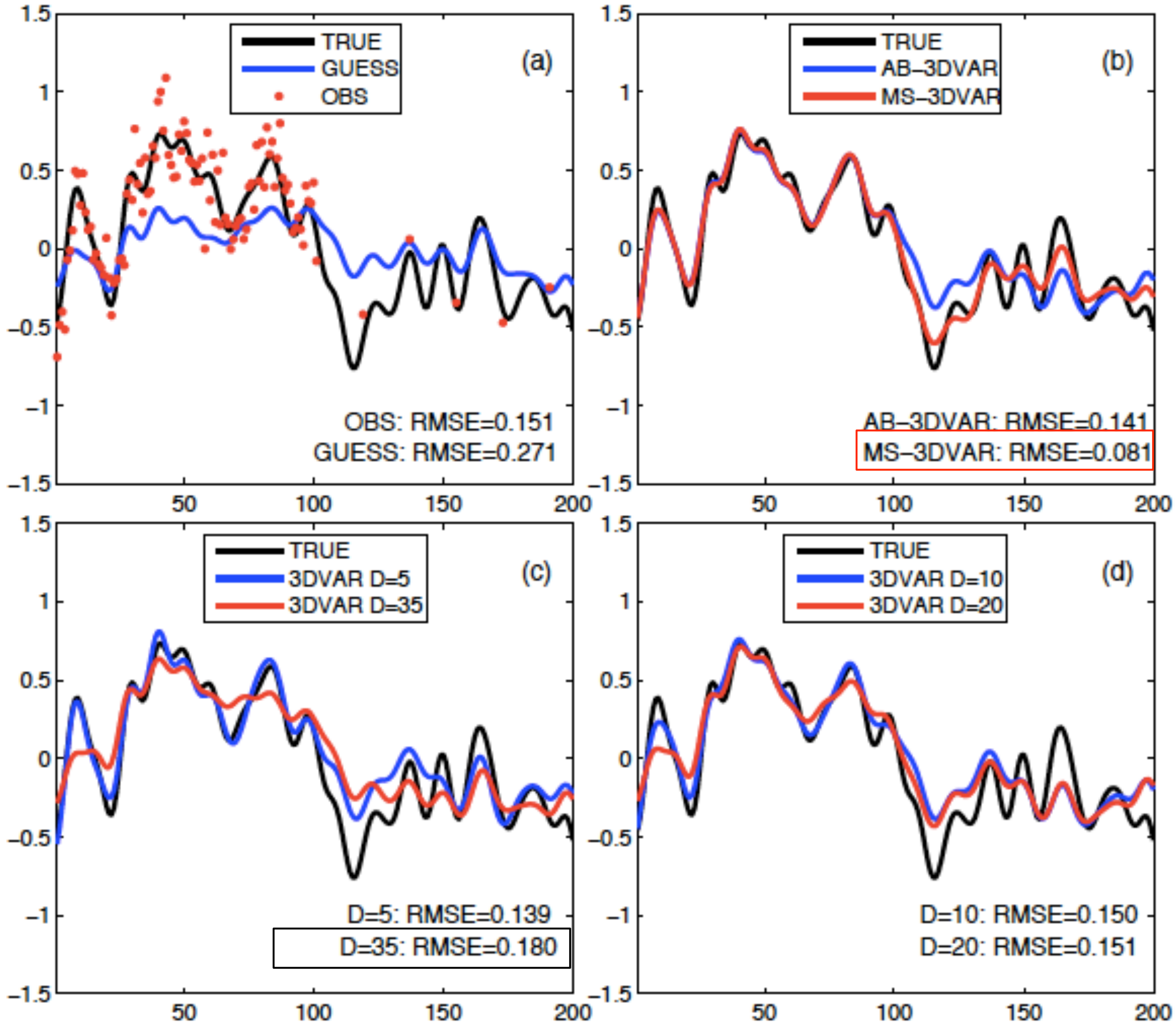


AB and MS work differently at D_S & D_L

MS captures LS more effectively than AB by sequential (successive) approach

Mixed Resolution Network

- Analysis ($\mathbf{x}^a$)



MS works much better than AB

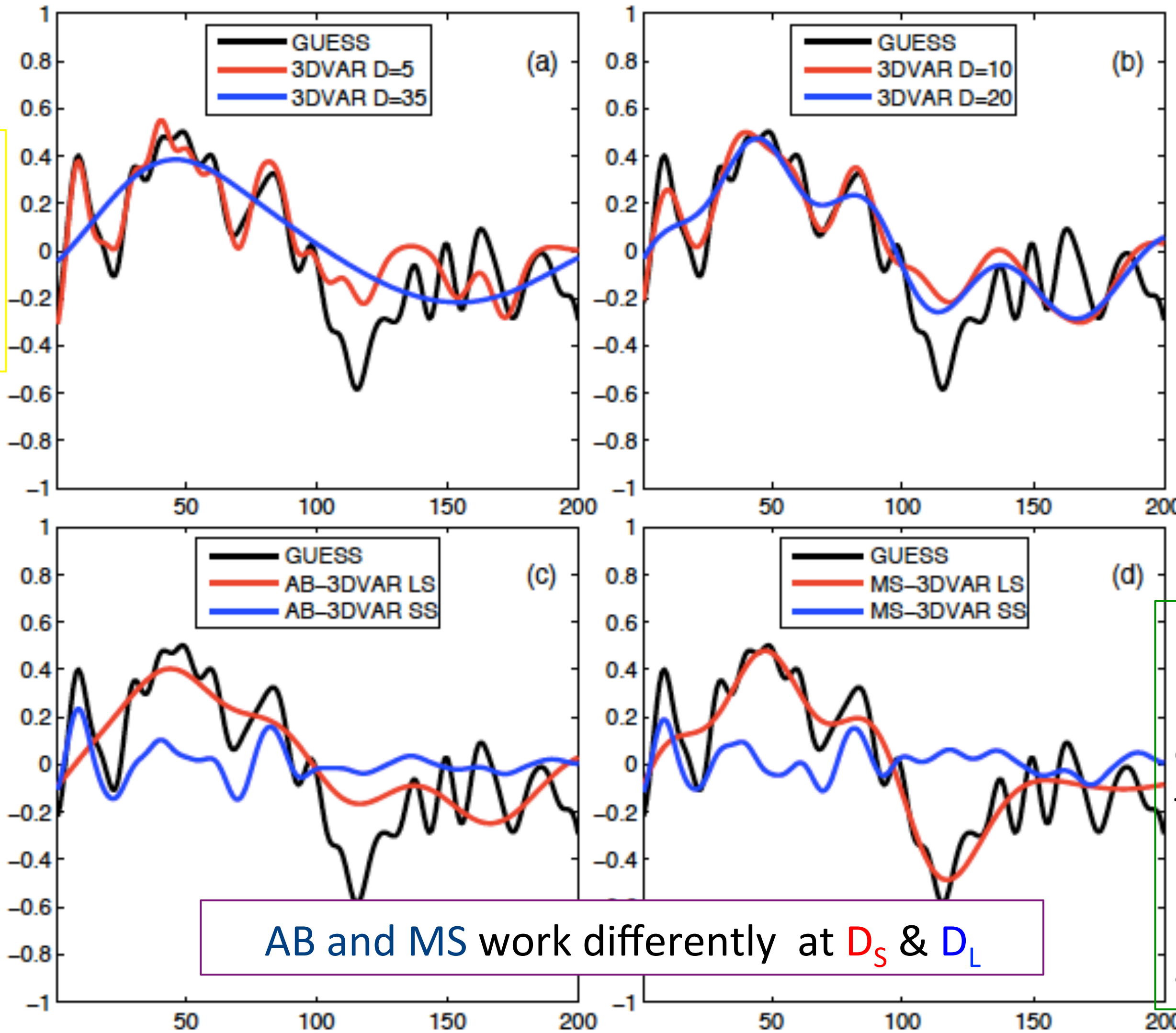
3V work OK: Better at D_S than D_L

3D work OK at medium D

Mixed Resolution Network

- Analysis Increment ($\Delta \mathbf{x}^a$)

3D don't work well: Better at D_S than D_L



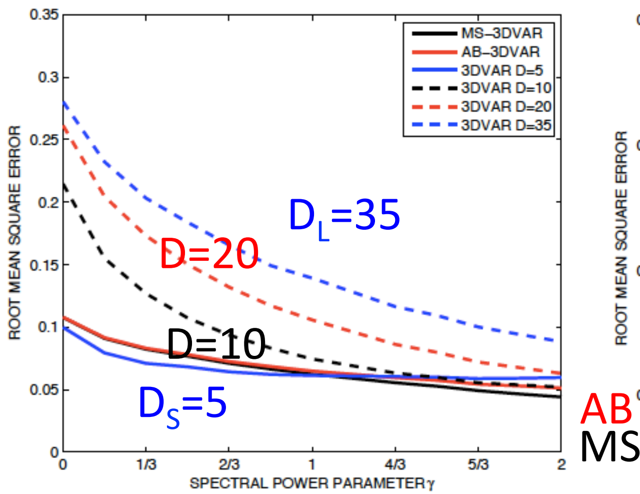
AB and MS work differently at D_S & D_L

MS captures LS more effectively than AB by sequential (successive) approach

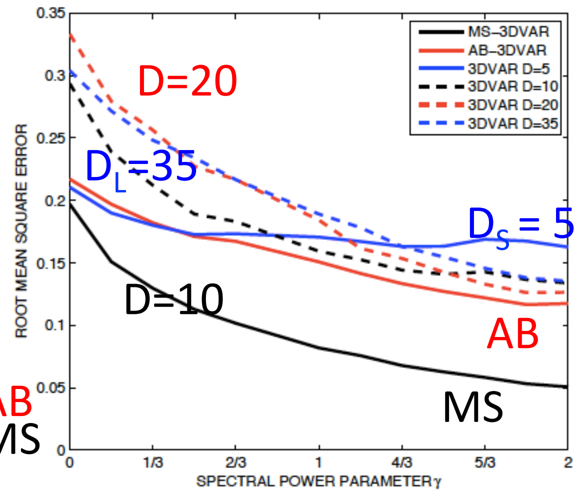
Scale-Dependence: Analysis RMSE

- Performance depends on treatment of MS in **B** (D) and **H**

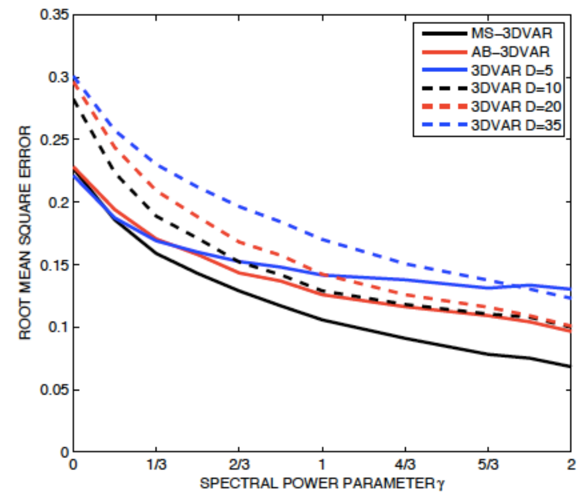
H: completely dense



H: patchy dense



H: mixed dense-coarse



larger scale

smaller scale

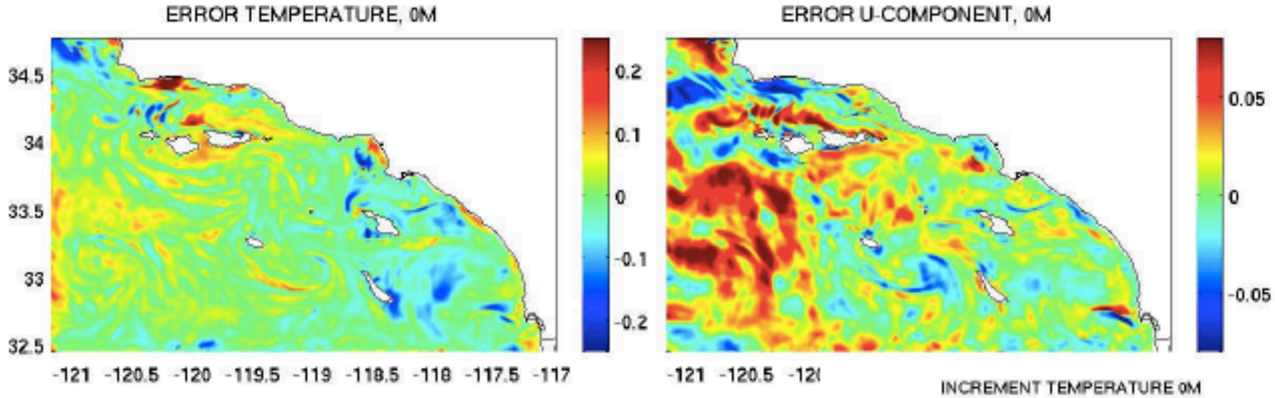
Concluding Remarks

- MS-MR data assimilation is formulated
  - Works very well!
  - Issue: Scale separation, particularly $\mathbf{y}_D$ & $\mathbf{R}_D$

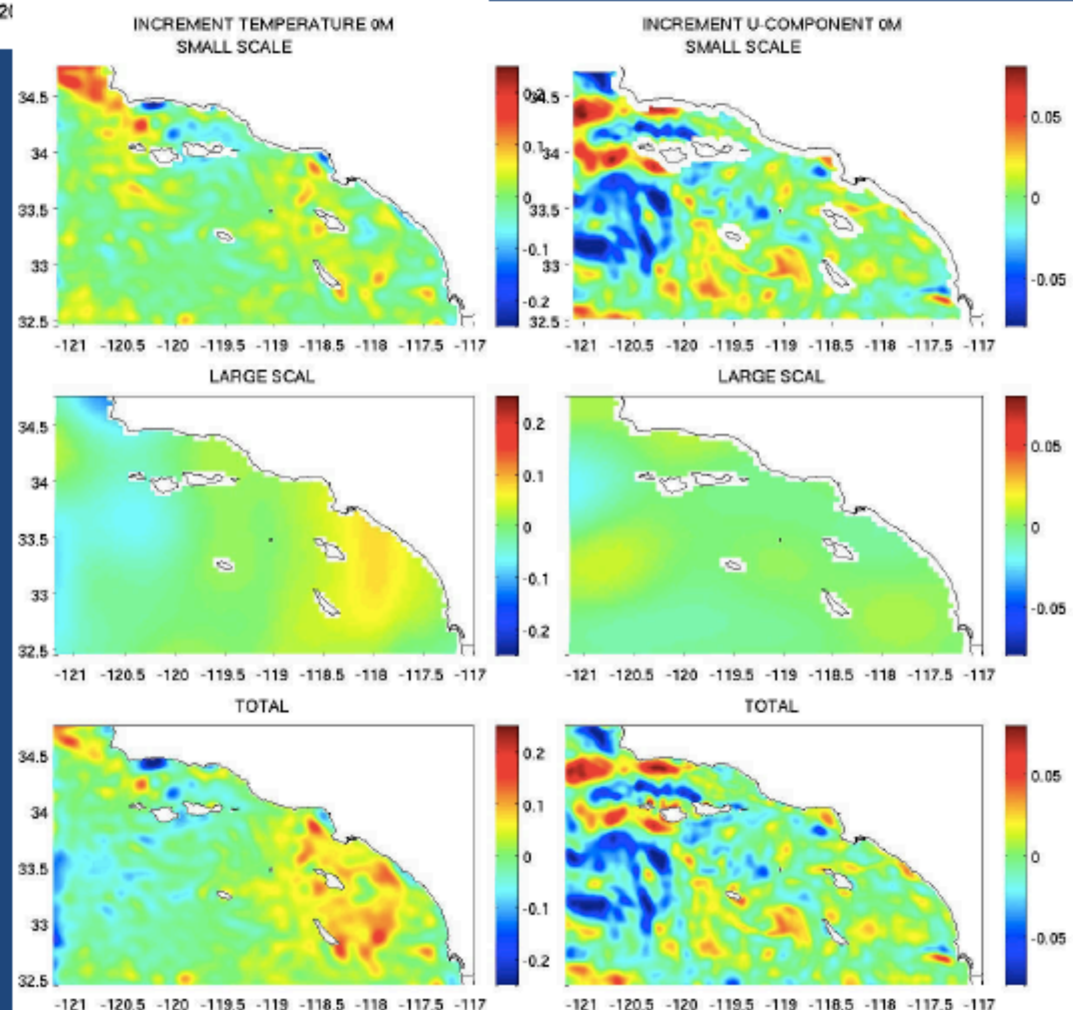
- Examples
  - SCB
    - OSSE
    - Real data experiments
  - Illustration using simple 1D example

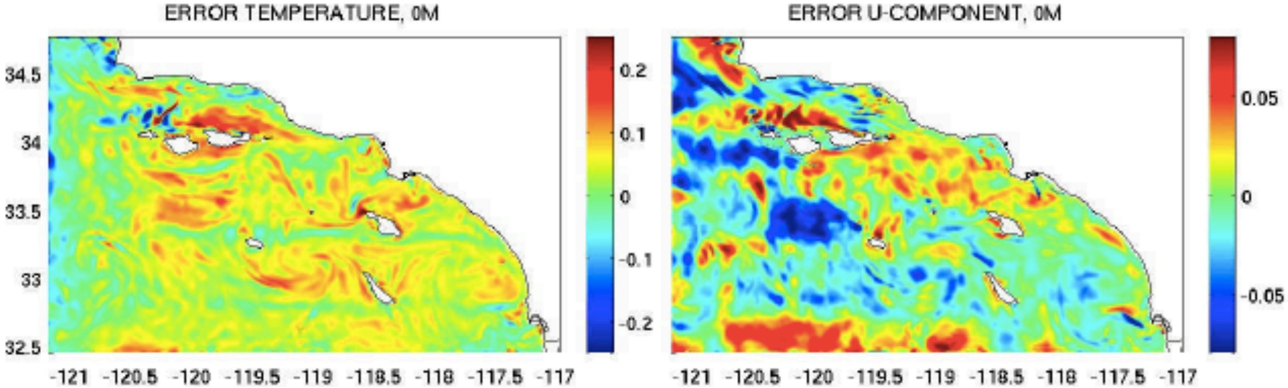
- Applications
  - MS3DVAR: developed for Southern California Bight (SCB)
  - MS-MR LETKF: being implemented into SCB & Lorenz MS model
  - Hybrid: developed & to be implemented in the SCB & Chesapeake Bay

Backup



UTC 21, 2008-01-01





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