Hybrid 4-D data assimilation with and without the tangent linear model

Jon Poterjoy and Fuqing Zhang

Department of Meteorology
The Pennsylvania State University

Monday 19^{th} May, 2014

Background: operational 3DVar and 4DVar

Variational data assimilation methods have been used operationally in NWP since the early 1990s:

3DVar: *Parrish and Derber (1992), Courtier et al. (1998), Lorenc et al. (2000)*

Given $\mathbf{x}_0 \sim N(\mathbf{x}_0^b, \mathbf{B})$ and $\mathbf{y}_0 \sim N(H_0(\mathbf{x}_0^{true}), \mathbf{R}_0)$, the posterior mean is the solution that minimizes:

$$\begin{aligned} J(\mathbf{x}_0) &= \tfrac{1}{2}(\mathbf{x}_0^b - \mathbf{x}_0)^T \mathbf{B}^{-1} (\mathbf{x}_0^b - \mathbf{x}_0) \\ &+ \tfrac{1}{2}[\mathbf{y}_0 - H_0(\mathbf{x}_0)]^T \mathbf{R}_0^{-1} [\mathbf{y}_0 - H_0(\mathbf{x}_0)]. \end{aligned}$$

Background: operational 3DVar and 4DVar

Variational data assimilation methods have been used operationally in NWP since the early 1990s:

4DVar: *Rabier et al. (2000), Gauthier and Thepáut (2001)*

Given $\mathbf{x}_0 \sim N(\mathbf{x}_0^b, \mathbf{B})$ and $\mathbf{y}_t \sim N(H_t(\mathbf{x}_t^{true}), \mathbf{R}_t)$, the posterior mean is the solution that minimizes:

$$\begin{aligned} J(\mathbf{x}_0) &= \tfrac{1}{2}(\mathbf{x}_0^b - \mathbf{x}_0)^T \mathbf{B}^{-1} (\mathbf{x}_0^b - \mathbf{x}_0) \\ &+ \tfrac{1}{2}\sum_{t=0}^{T} \{\mathbf{y}_t - H_t[M_t(\mathbf{x}_0)]\}^T \mathbf{R}_t^{-1} \{\mathbf{y}_t - H_t[M_t(\mathbf{x}_0)]\}. \end{aligned}$$

Background: operational 3DVar and 4DVar

Variational data assimilation methods have been used operationally in NWP since the early 1990s:

Incremental 4DVar: *Courtier et al. (1994)*

Use linear operators to transform increments at $t = 0$ into observation space at $t = \tau$:

$$\begin{aligned} H_\tau[M_\tau(\mathbf{x}_0)] &= H_\tau[M_\tau(\mathbf{x}_0^b + \delta\mathbf{x}_0)], \\ &\approx H_\tau[M_\tau(\mathbf{x}_0^b)] + \mathbf{H}_\tau \mathbf{M}_\tau \delta\mathbf{x}_0. \end{aligned}$$

Cost function is minimized with respect to $\delta\mathbf{x}_0$.

Background: ensemble data assimilation

Ensemble Kalman filters emerged as an alternative to 3DVar and 4DVar in the late 1990s and 2000s.

Stochastic filter: *Evensen (1994),*
Houtekamer and Mitchell (1998)

Deterministic filter: *Bishop et al. (2001),*
Anderson (2001),
Whitaker and Hamill (2002)

Background: hybrid data assimilation

Hamill and Snyder (2000) used a combination of static and ensemble error covariance in a hybrid EnKF-3DVar scheme:

$$\mathbf{B} \;=\; \beta \mathbf{B}_s + (1-\beta)\mathbf{B}_e.$$

Hybrid analyses use flow-dependent information, but are less sensitive to ensemble size than EnKF.

Etherton and Bishop (2004) showed that the $\mathbf{B}_s$ component also helps reduce bias associated with model error in EnKF.

Background: hybrid data assimilation

Lorenc (2003) introduced extended control variables as a means of using localized ensemble information in operational variational methods with preconditioning.

Buehner (2005) examined the impact of introducing ensemble information in an operational 3DVar system.

Wang et al. (2008a,b) developed and tested a hybrid ETKF-3DVar for a limited-area model.

Background: 4DVar with hybrid **B**

"4-D-Var or ensemble Kalman filter?" *(Kalnay et al. 2009)*

Why not both?

Zhang et al. (2009) proposed a hybrid EnKF-4DVar system and tested with Lorenz (1996) model.

Buehner et al. (2010a,b) compared various forms of 3-D and 4-D ensemble variational methods in an operational model.

Zhang and Zhang (2012) implemented hybrid EnKF-4DVar in a limited-area modeling framework.

Background: 4D-ensemble-var

Liu et al. (2008,2009) introduced a variational scheme that uses a 4-D ensemble to replace tangent linear and adjoint model operators in 4DVar (similar to 4-D EnKF).

Ensemble 4DVar with and without the tangent linear model are compared for operational DA in *Buehner et al. (2010a,b)*.

Fairbairn et al. (2014) provide a more systematic evaluation of various ensemble 4DVar methods in a simple-model framework.

Where are we today?

There are two major types of hybrid four-dimensional data assimilation methods for operational NWP and research.

E4DVar: 4DVar with a mix of ensemble and static background error covariance.

4DEnVar: E4DVar with linearized model operations replaced by 4-D ensemble.

E4DVar and 4DEnVar

The primary difference between E4DVar and 4DEnVar is how covariance is approximated in the assimilation window.

For linearly growing errors, analysis increments depend implicitely on:

$$\overline{\mathbf{x}_0'[H_\tau(\mathbf{x}_\tau)']^T} \leftarrow \textit{Covariance between } \mathbf{x}_0 \textit{ and } \mathbf{x}_\tau \textit{ in observation space.}$$

$$\overline{H_\tau(\mathbf{x}_\tau)'[H_\tau(\mathbf{x}_\tau)']^T} \leftarrow \textit{Covariance of } \mathbf{x}_\tau \textit{ in observation space.}$$

E4DVar and 4DEnVar

E4DVar:

$$\begin{aligned}
\overline{\mathbf{x}_0'[H_\tau(\mathbf{x}_\tau)']^T} &\approx [\mathbf{H}_\tau\mathbf{M}_\tau\mathbf{B}]^T \\
&= \{\mathbf{H}_\tau\mathbf{M}_\tau[\beta\mathbf{B}_s + (1-\beta)\mathbf{B}_e]\}^T \\
&= \beta[\mathbf{H}_\tau\mathbf{M}_\tau\mathbf{B}_s]^T + (1-\beta)[\mathbf{H}_\tau\mathbf{M}_\tau\mathbf{B}_e]^T
\end{aligned}$$

Uses $\mathbf{M}_\tau$ and $\mathbf{H}_\tau$ to approximate time-dependent covariance, starting from static covariance $\mathbf{B}_s$ and ensemble covariance $\mathbf{B}_e$.

E4DVar and 4DEnVar

4DEnVar:

$$\overline{\mathbf{x}_0'[H_\tau(\mathbf{x}_\tau)']^T} \approx \beta[\mathbf{H}_\tau \mathbf{B}_s]^T + (1-\beta)\Big(\tfrac{1}{N_e-1}\sum_{n=1}^{N_e} \mathbf{x}_{0,n}'[H_\tau(\mathbf{x}_{\tau,n})']^T\Big)$$

Uses nonlinear operators to estimate ensemble covariance, but cannot evolve $\mathbf{B}_s$ in time.

Data assimilation at PSU

Our research group requires accurate, efficient data assimilation methods for mesoscale weather applications.

Examples from this workshop

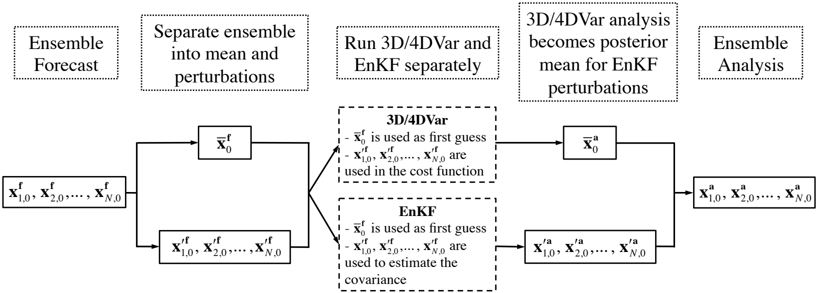
Yunji Zhang: *Ensemble Data Assimilation for the 2013 supercell event with an F5 tornado over Moore, Oklahoma*

Ashford Reyes: *Assimilation of conventional and field experiment observations for Hurricane Mathew during NASA GRIP 2010*

Yonghui Weng and Fuqing Zhang: *Advances in tropical cyclone inner-core data assimilation, overview*

Data assimilation at PSU

Current strategy: a two-way coupled EnKF-Var hybrid data assimilation for the WRF model, using E4DVar or 4DEnVar to estimate posterior mean.



(Zhang and Zhang 2012, Poterjoy and Zhang 2014)

Analysis increments

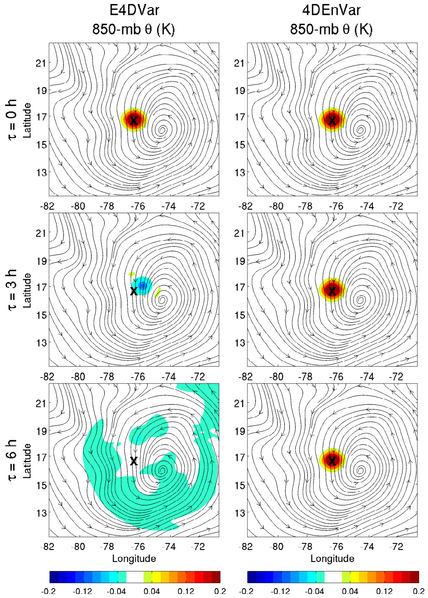
E4DVar and 4DEnVar single-observation experiments

Single-observation experiments provide a means of examining the structure of $\overline{\mathbf{x}_0'[H_\tau(\mathbf{x}_\tau)']^T}$.

850-mb temperature observations are created at $\tau = 0$, 3 and 6 h by adding 1 K to temperature values from $\mathbf{x}_t^b$.

850-mb θ analysis increments at $t = 0$ are shown for varying weights of $\mathbf{B}_s$ and $\mathbf{B}_e$.

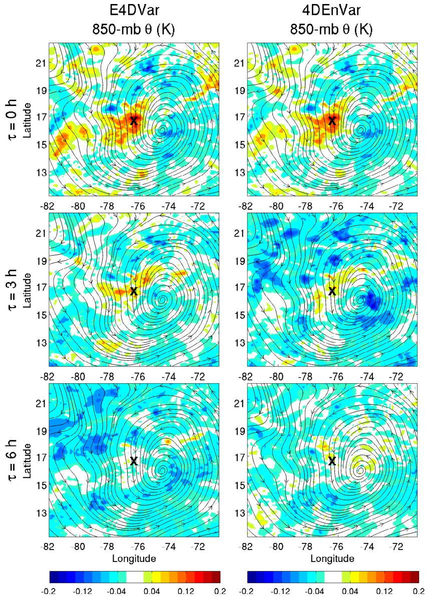
0% ensemble



With no ensemble information:

- E4DVar becomes 4DVar
- 4DEnVar becomes 3DVar with innovations calculated at correct observation times

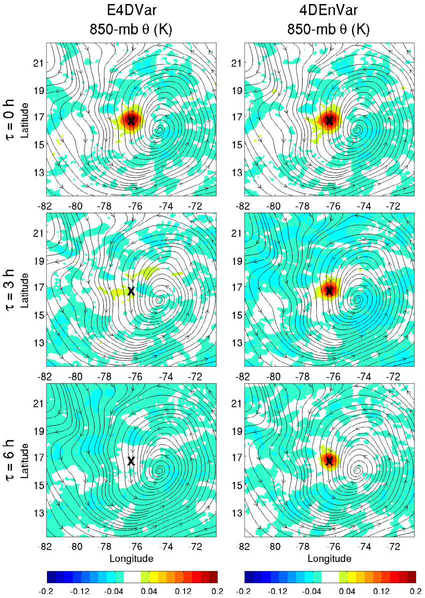
100% ensemble



The two methods are theoretically equivalent for $\tau = 0$ and $\mathbf{B} = \mathbf{B}_e$.

Significant differences occur for $\tau > 0$, owing to covariance localization, and assumptions made in forming $\mathbf{M}_\tau$.

50% ensemble



With 50% of the covariance coming from $\mathbf{B}_e$, 4DEnVar increments remain largely isotropic near observations.

Questions

How do the two methods respond to changes in ensemble size, window length, covariance localization, inflation, hybrid weights, and observing system?

What is the significance of not using $\mathbf{M}_t$ to evolve $\mathbf{B}_s$ in 4DEnVar?

What are the largest practical challenges for applying the two methods for mesoscale data assimilation?

Experiments with a simple model

The Lorenz (1996) model is used to compare E4DVar and 4DEnVar:

$$\frac{dx_i}{dt} = (x_{i+1} - x_{i-2})x_{i-1} - x_i + F.$$

Experiment details:

- 40 variables
- Observations of half the model variables are created every 1.2 h from a "truth" simulation with added noise: $\epsilon_t \sim N(0, I)$.
- Analysis mean RMSEs are averaged over 2500 days in each experiment after a 100-day spin-up period.

Experiments with a simple model

Cycling E4DVar and 4DEnVar systems are coupled with EnKF using the same algorithm applied for the WRF model.

$\mathbf{B}_s$ is estimated from climatology for each configuration of the model, data assimilation and observing system.

$\mathbf{B}_e$ is localized using Gaussian function, with tunable radius of influence (ROI).

Posterior ensemble variance is inflated by relaxing posterior perturbations to prior perturbations *(Zhang et al 2004)*:

$$\mathbf{x}'^{a}_{t,n} \;\leftarrow\; (1-\alpha)\mathbf{x}'^{a}_{t,n} + \alpha\mathbf{x}'^{f}_{t,n}.$$

Ensemble Size

Results with $T = 6$ h, full ensemble covariance, and varying N_e, *ROI*, and α.

E4DVar outperforms 4DEnVar when localization is needed for small ensembles.

E4DVar

Ne = 5

ROI \ α	0.3	0.4	0.5	0.6	0.7	0.8
1	0.313	0.259	0.263	0.270	0.301	0.402
2	NA	0.273	**0.235**	0.236	0.258	0.332
3	NA	NA	NA	0.308	0.244	0.297
4	NA	NA	NA	NA	0.498	0.298
5	NA	NA	NA	NA	NA	0.333
6	NA	NA	NA	NA	NA	NA

Ne = 10

ROI \ α	0.2	0.3	0.4	0.5	0.6	0.7
2	0.197	0.193	0.196	0.203	0.219	0.250
3	NA	0.177	0.179	0.184	0.198	0.227
4	NA	0.178	0.171	0.175	0.188	0.215
5	NA	0.176	**0.169**	0.176	0.182	0.207
6	NA	NA	NA	0.174	0.180	0.202
7	NA	NA	NA	0.189	0.182	0.202

Ne = 20

ROI \ α	0.2	0.3	0.4	0.5	0.6	0.7
4	0.162	0.162	0.165	0.172	0.188	0.219
5	0.164	0.158	0.160	0.167	0.182	0.212
6	0.157	0.156	0.157	0.164	0.178	0.206
7	NA	**0.154**	0.156	0.163	0.176	0.204
8	NA	0.155	0.157	0.162	0.175	0.202
9	NA	NA	0.158	0.161	0.174	0.201

Ne = 40

ROI \ α	0	0.1	0.2	0.3	0.4	0.5
Inf	NA	NA	**0.131**	0.131	0.136	0.145

4DEnVar

Ne = 5

ROI \ α	0.3	0.4	0.5	0.6	0.7	0.8
1	0.402	0.371	0.319	0.299	0.339	0.480
2	NA	NA	0.302	**0.254**	0.278	NA
3	NA	NA	NA	0.421	0.276	0.324
4	NA	NA	NA	NA	0.325	0.319
5	NA	NA	NA	NA	NA	0.529
6	NA	NA	NA	NA	NA	NA

Ne = 10

ROI \ α	0.2	0.3	0.4	0.5	0.6	0.7
2	0.211	0.205	0.206	0.211	0.229	0.262
3	NA	0.187	0.186	0.190	0.205	0.233
4	0.210	NA	0.182	0.181	0.195	0.219
5	NA	NA	0.186	**0.176**	0.188	0.212
6	NA	NA	0.177	0.179	0.189	0.207
7	NA	NA	NA	0.347	0.189	0.205

Ne = 20

ROI \ α	0.2	0.3	0.4	0.5	0.6	0.7
4	0.168	0.166	0.169	0.176	0.192	0.224
5	0.163	0.161	0.164	0.171	0.186	0.217
6	NA	**0.160**	0.162	0.168	0.182	0.211
7	NA	NA	0.161	0.166	0.180	0.208
8	NA	NA	0.161	0.166	0.179	0.205
9	NA	NA	0.166	0.165	0.178	0.209

Ne = 40

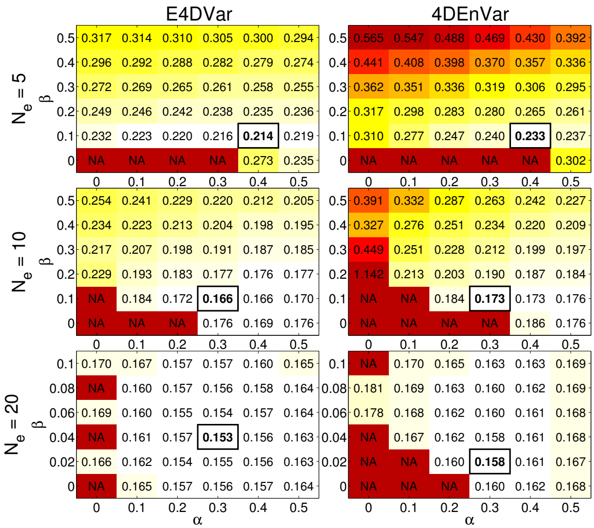
ROI \ α	0	0.1	0.2	0.3	0.4	0.5
Inf	NA	NA	**0.131**	0.132	0.137	0.146

Mean RMSE for ranges of α and *ROI*.

Hybrid covariance

Results with $T = 6$ h, fixed ROI, and varying N_e, β, and α.

Hybrid covariance produces slightly more accurate analyses, but relaxation remains dominant mechanism for treating sampling errors.

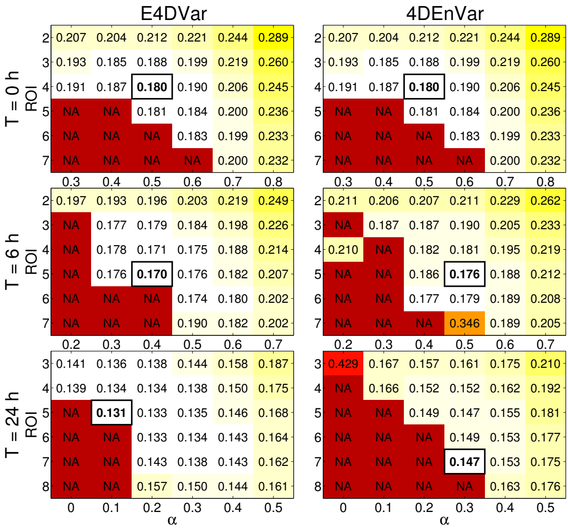


Mean RMSE for ranges of β and α.

Window length

Results with full ensemble covariance, $N_e = 10$, and varying T, β, and α.

E4DVar improves more with window length than 4DEnVar.

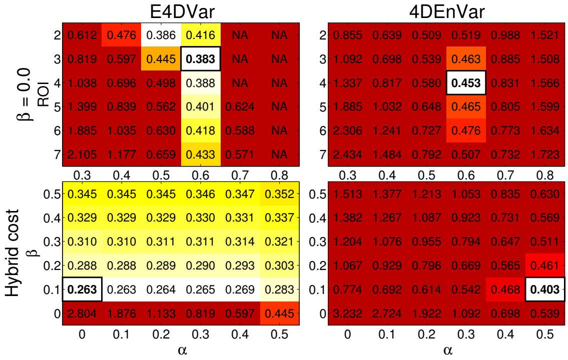


Mean RMSE for ranges of α and *ROI*.

Model error

Full ensemble covariance (top) for $N_e = 20,\ T = 24$ h and $F_m = 7$.

ROI is fixed (bottom) and sensitivity to β is shown.

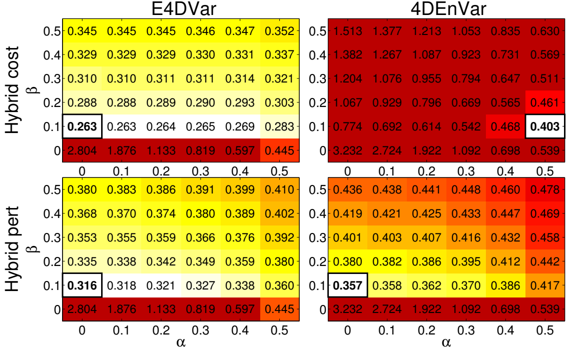


Mean RMSE for ranges of α and *ROI*.

Hybrid covariance is effective at treating model error in E4DVar—it also reduces the optimal α to near zero.

Model error

Ensemble forecast perturbations are combined with samples drawn from $N(0, \mathbf{B}_s)$ (similar to additive inflation).



Mean RMSE for ranges of α and *ROI*.

E4DVar performs slightly worse than hybrid cost function case because of extra sampling errors, but 4DEnVar benefits from propagating $\mathbf{B}_s$ forward.

4-D covariance

For $\mathbf{B} = \mathbf{B}_e$ with no localization, E4DVar is the same as 4DEnVar, but with linear approximations for H_τ and M_τ.

E4DVar:

$$\begin{aligned} \overline{\mathbf{x}_0'[H_\tau(\mathbf{x}_\tau)']^T} &\approx [\mathbf{H}_\tau \mathbf{M}_\tau \mathbf{B}_e]^T \\ &= \tfrac{1}{N_e - 1} \sum_{n=1}^{N_e} \mathbf{x}_{0,n}' [\mathbf{H}_\tau \mathbf{M}_\tau \mathbf{x}_{0,n}']^T \end{aligned}$$

4DEnVar:

$$\overline{\mathbf{x}_0'[H_\tau(\mathbf{x}_\tau)']^T} \approx \tfrac{1}{N_e - 1} \sum_{n=1}^{N_e} \mathbf{x}_{0,n}' [H_\tau(\mathbf{x}_{\tau,n})']^T$$

4-D covariance

Localization is required for practical applications, and it is accomplished differently in the two methods.

E4DVar:

$$\overline{\mathbf{x}_0'[H_\tau(\mathbf{x}_\tau)']^T} \approx [\mathbf{H}_\tau \mathbf{M}_\tau (\mathbf{B}_e \circ \rho)]^T$$

4DEnVar:

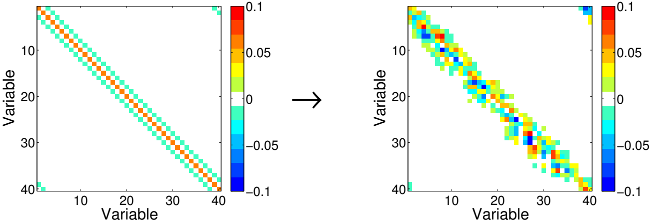
$$\overline{\mathbf{x}_0'[H_\tau(\mathbf{x}_\tau)']^T} \approx \left(\tfrac{1}{N_e - 1} \sum_{n=1}^{N_e} \mathbf{x}_{0,n}'[H_\tau(\mathbf{x}_{\tau,n})']^T \right) \circ \rho$$

4-D covariance example

10 samples are drawn from $N(\mathbf{x}_0^b, \mathbf{B}_s)$ to estimate $\mathbf{P}_{0,\tau}$ with and without localization, where

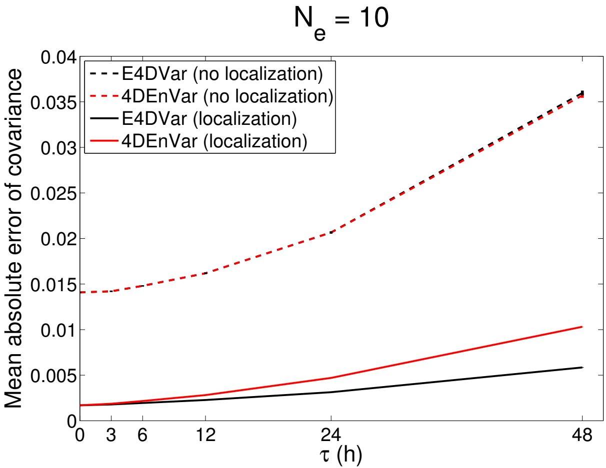
$$\mathbf{P}_{0,\tau} = \overline{\mathbf{x}_0' \mathbf{x}_\tau'^T}.$$

$\mathbf{B}_s$ $\qquad\qquad$ $\mathbf{P}_{0,\tau}^{truth}$



True $\mathbf{P}_{0,\tau}$, estimated from a 10^5-member ensemble for $\tau = 24$ h.

Results from 1000 trials

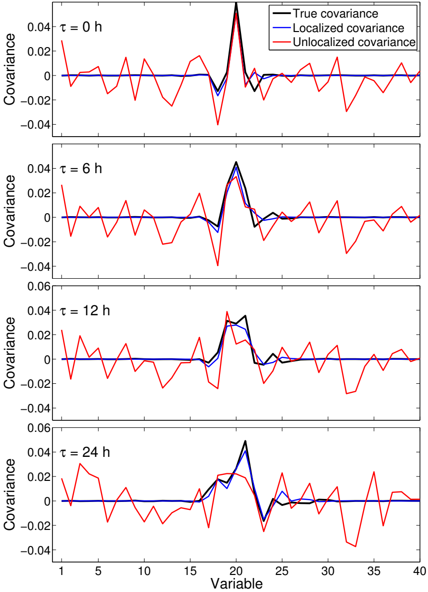


E4DVar outperforms 4DEnVar because of better localization!

Spatio-temporal covariance of variable 20

Column 20 of $\mathbf{P}_{0,\tau}$ is plotted with and without $\mathbf{B}_e$ localization at $t = 0$.

Without localization at $t = 0$, the resulting $\mathbf{P}_{0,\tau}$ requires both localization and variance inflation downstream of variable 20.



Summary

Two hybrid 4-D data assimilation methods are considered for mesoscale applications using the WRF model.

Idealized experiments show that E4DVar outperforms 4DEnVar because of better localization and the ability to propagate $\mathbf{B}_s$ in time.

An example using $\mathbf{M}_t$ to examine the 4-D localization needed by 4DEnVar to achieve the same results as E4DVar demonstrates a major challenge for data assimilation methods that use 4-D ensembles.

Outstanding questions

PENNSTATE

What are the impacts of nonlinearity and simplified physical parameterization schemes in $\mathbf{M}_t$ for data assimilation at cloud-permitting scales?

How well can E4DVar and 4DEnVar assimilate time-integrated quantities that relate nonlinearly to the model state?

Which method is more effective for mesoscale applications?