Ensemble Singular Vectors and their use as additive inflation in EnKF

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Introduction

- Ensemble-based leading singular vectors (ESVs) indicates the directions of the fastest growing forecast errors
  - Given the choice of the perturbation norm and forecast interval, the leading ESV maximizes the growth of the perturbations.
- Additive covariance inflation aims to perturb the subspace spanned by the ensemble vectors and better capture the sub-growing directions that may be missed in the original ensemble.
  - Random perturbations may introduce non-growing or irrelevant error structures.
  - ESVs can be applied as a “flow-dependent” additive inflation to enhance structures of the background error covariance.

Ensemble-based singular vector (ESV)

A set of initial (I) and final (F) perturbations:

$$\mathbf{X}^{I}_{t-\Delta t} = \left[\delta\mathbf{x}_{1,t-\Delta t}, \cdots, \delta\mathbf{x}_{i,t-\Delta t}, \cdots, \delta\mathbf{x}_{K,t-\Delta t}\right]; \quad \mathbf{X}^{F}_{t} = \left[\delta\mathbf{x}^{F}_{1,t}, \cdots, \delta\mathbf{x}^{F}_{i,t}, \cdots, \delta\mathbf{x}^{F}_{K,t}\right]$$

Find the linear combination of initial perturbations that will grow fastest given a optimization time period (Δt)

Initial ES: $\delta\mathbf{x}^{I}_{t-\Delta t} = \mathbf{X}^{I}_{t-\Delta t}\mathbf{p}$

Final ES: $\delta\mathbf{x}^{F}_{t} = \mathbf{X}^{F}_{t}\mathbf{p}$

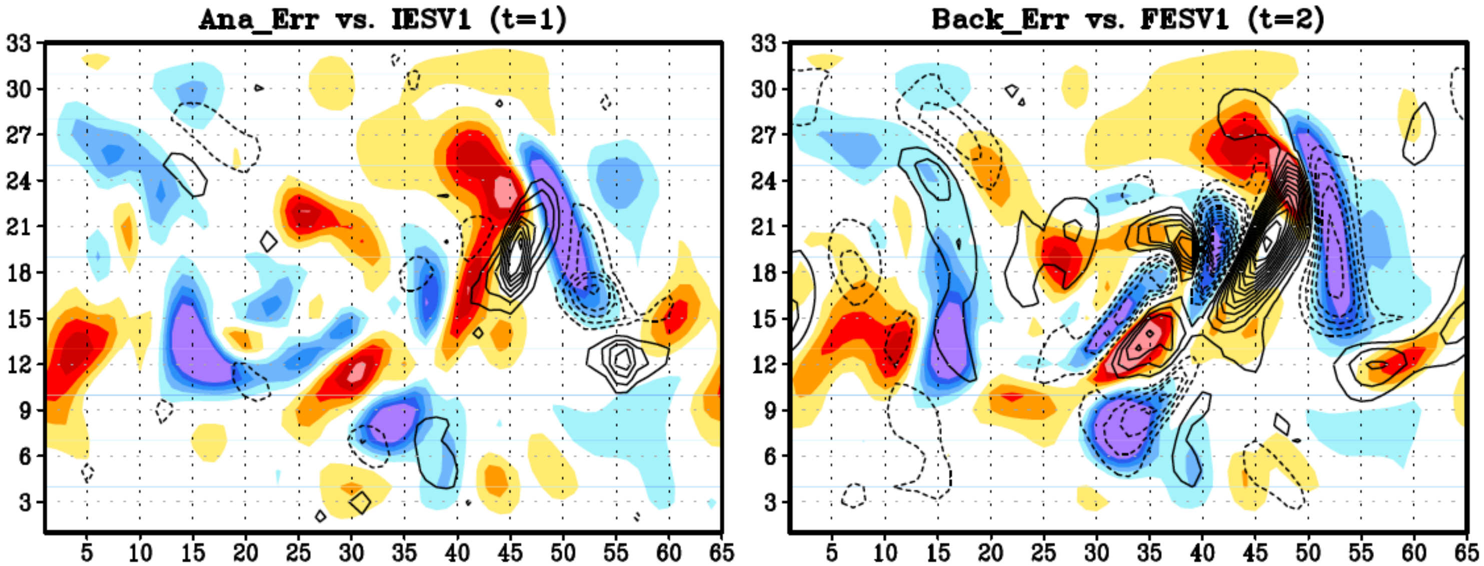
By defining the initial and final perturbation norms ($\mathbf{C}_I$ and $\mathbf{C}_F$), we can solve for the weights $\mathbf{p}$ (Enomoto et al. 2006).

$$\boxed{\left(\mathbf{X}^{I\,T}_{t-\Delta t}\,\mathbf{C}_I\,\mathbf{X}^{I}_{t-\Delta t}\right)^{-1}\left(\mathbf{X}^{F\,T}_{t}\,\mathbf{C}_F\,\mathbf{X}^{F}_{t}\right)\mathbf{p} = \lambda\,\mathbf{p}}$$

We can find K sets of IES and FES with $\left(\lambda^{i}, \mathbf{p}^{i} \quad i = 1, \cdots, K\right)$

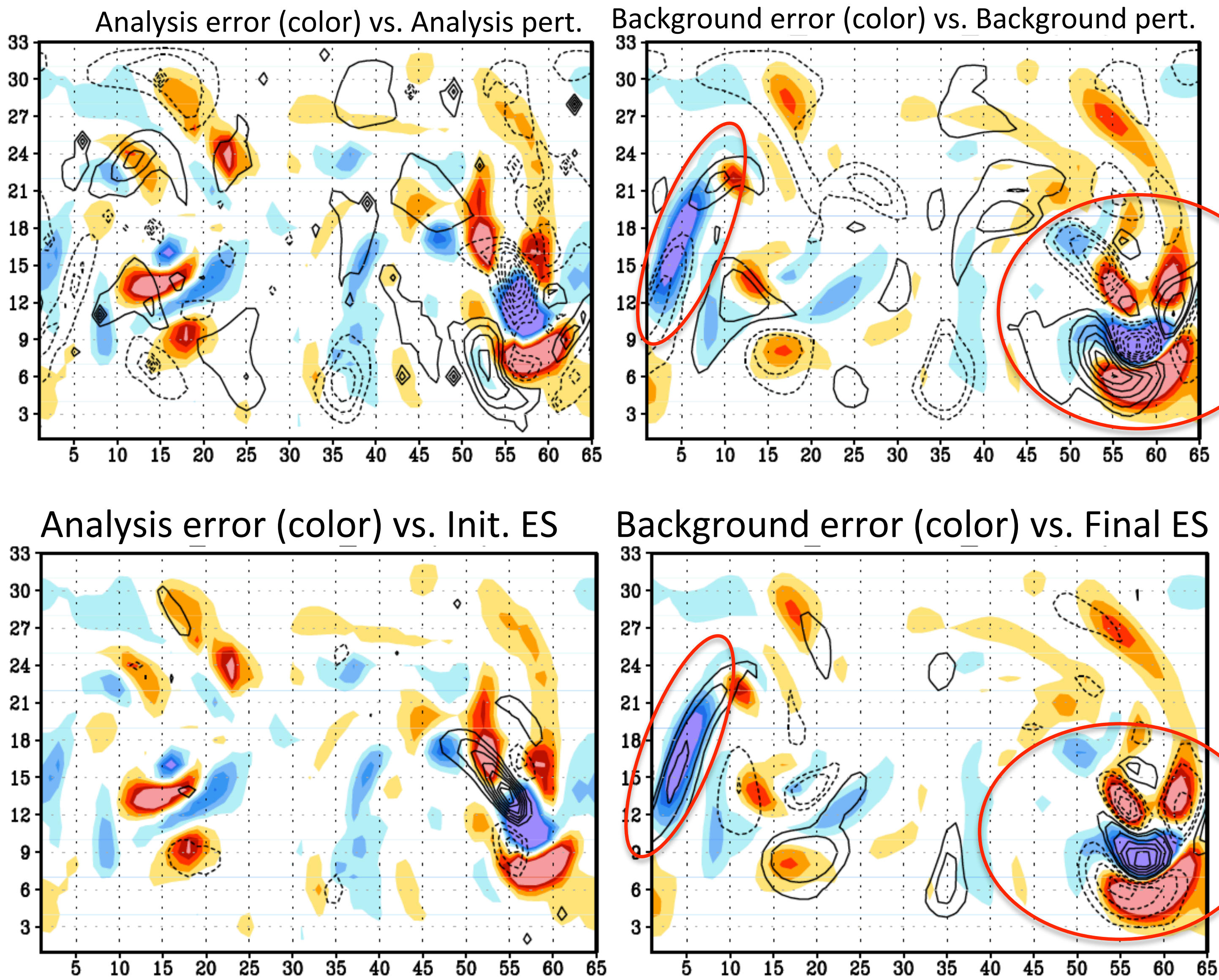
ESV_1 in a Quasi-geostrophic model

$\mathbf{X}^{I}_{t-\Delta t} = \mathbf{X}^{a}_{t-\Delta t}$ (LETKF Ana. Ens); $\mathbf{X}^{F}_{t} = \mathbf{X}^{b}_{t}$ (LETKF Back. Ens)



The fast growing perturbation (contours) is very closely related to the background errors (color). The IESV (an initial Singular Vector) is NOT related to the initial errors.

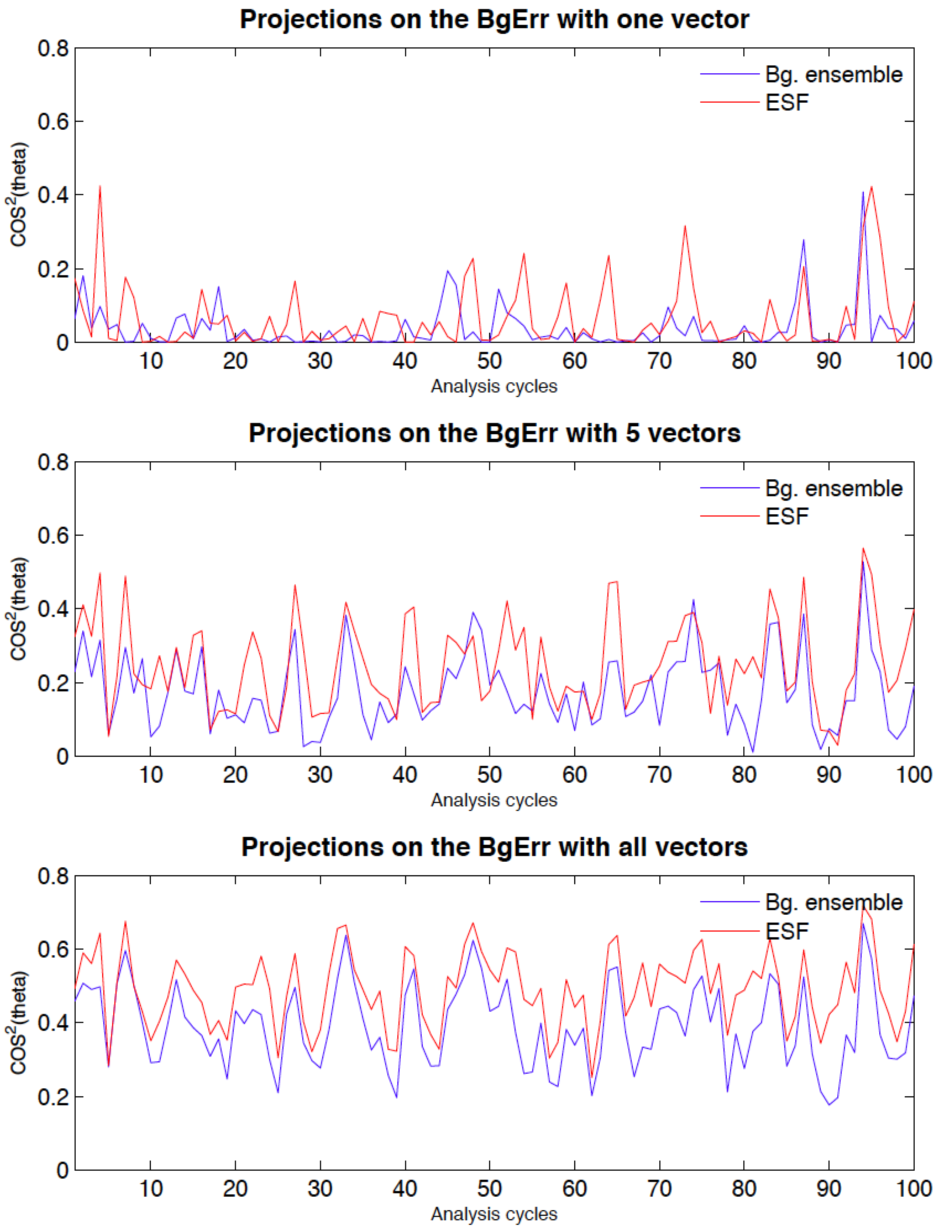
Comparisons between the ES, background ensemble perturbations and background errors



Background ensemble perturbations are still under development during LETKF’s spin-up

With the same ensemble subspace, the final ES effectively captures the fast growing errors.

Correlation with the background error



Compared to one ensemble member, the dominant final FESV generally projects better on the background errors.

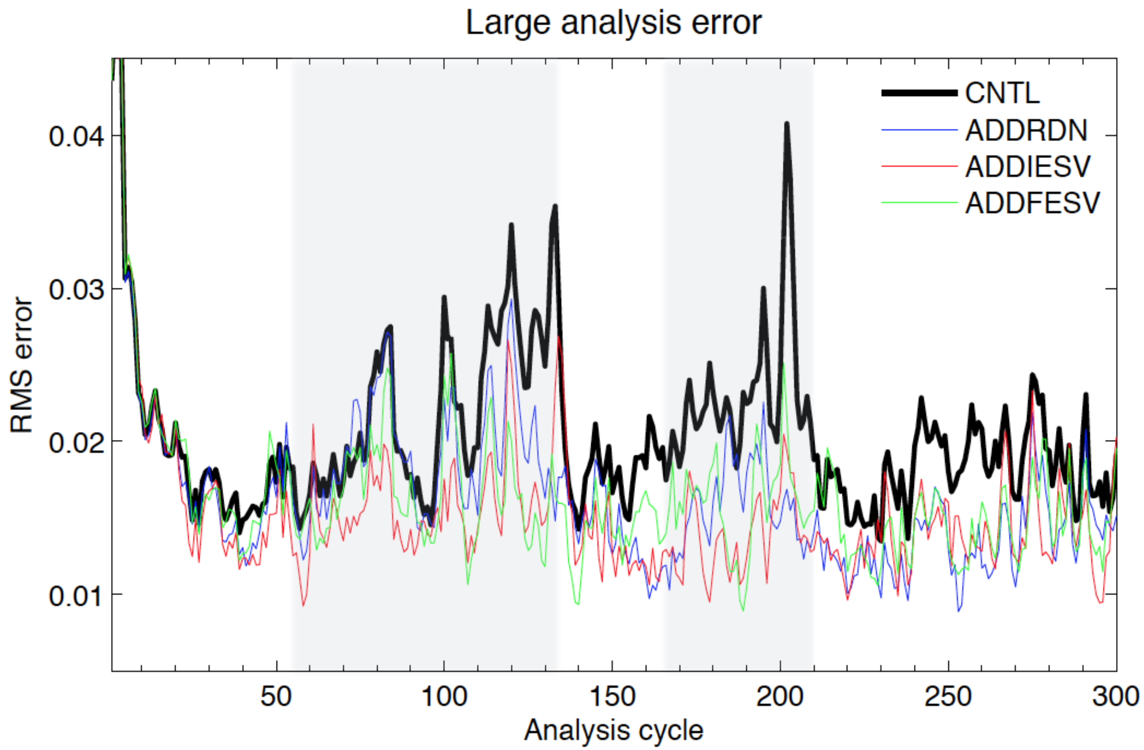
The subspace spanned by the final ESs can better represent the subspace of background errors.

Short summary

- Results suggest that the ESV can better represent the fast growing modes shown in the background errors.
- Apply ESV as the "flow-dependent" additive covariance inflation.
  - Perturbing the LETKF analysis ensemble with ESVs.

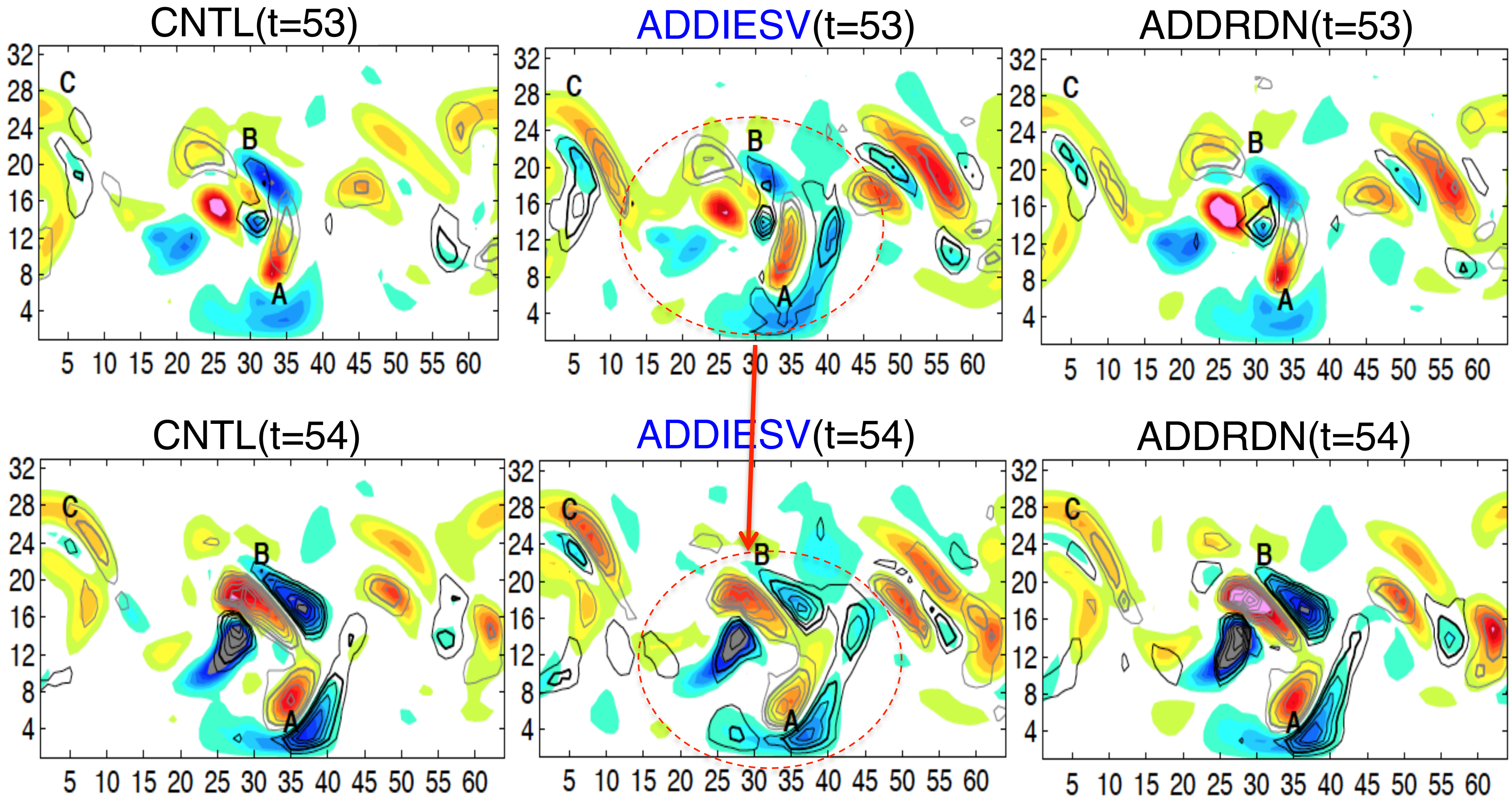
Analysis/Forecast error with additive inflation

CNTL: standard LETKF with multiplicative inflation
ADDRDN: LETKF with random perturbations as additive inflation
ADDIESV: LETKF with IESVs as additive inflation
ADDFESV: LETKF with FESVs as additive inflation

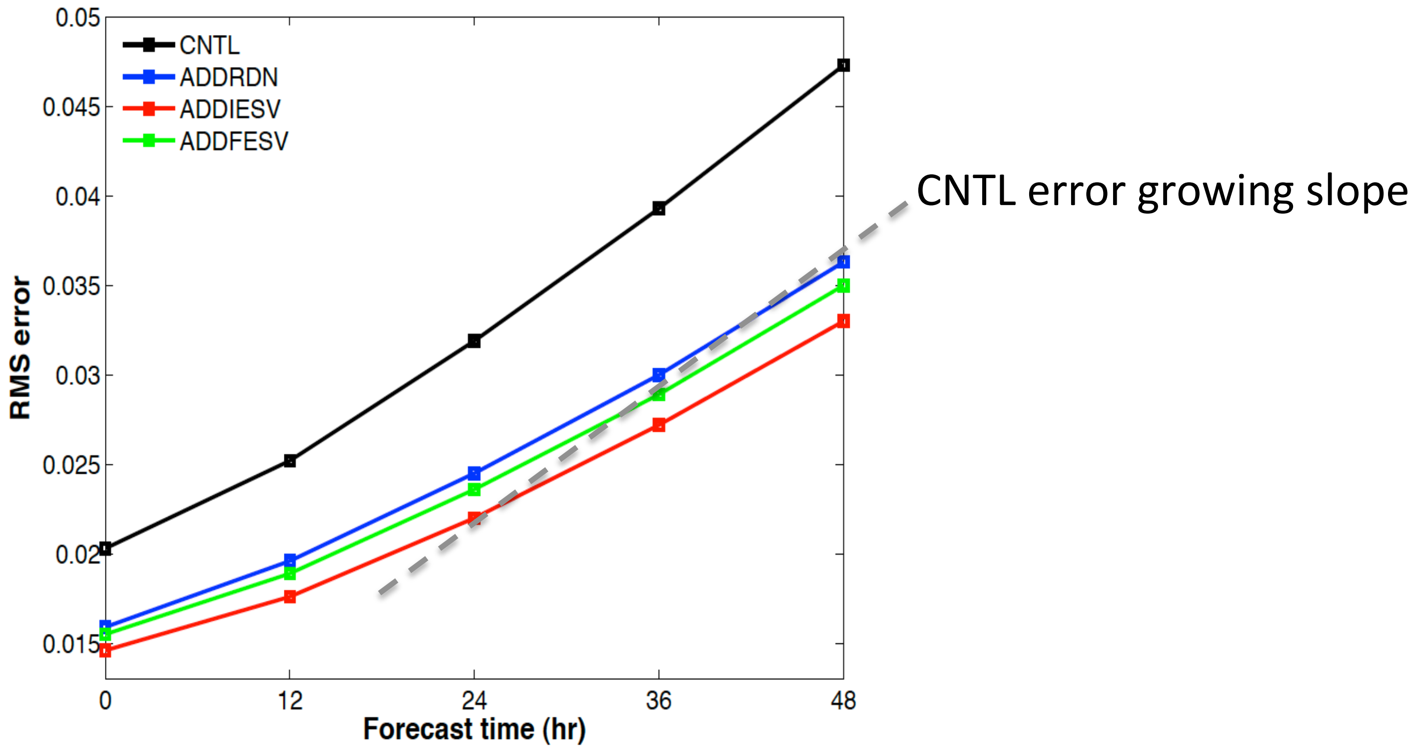


ADDIESV is particularly effective in correcting fast growing errors!!

Background error (color) vs. analysis increment (contour)



Forecast errors in time



- In CNTL, the incompletely removed growing errors **amplify** at later forecast time.
- ADDIESV successfully reduces growing errors!

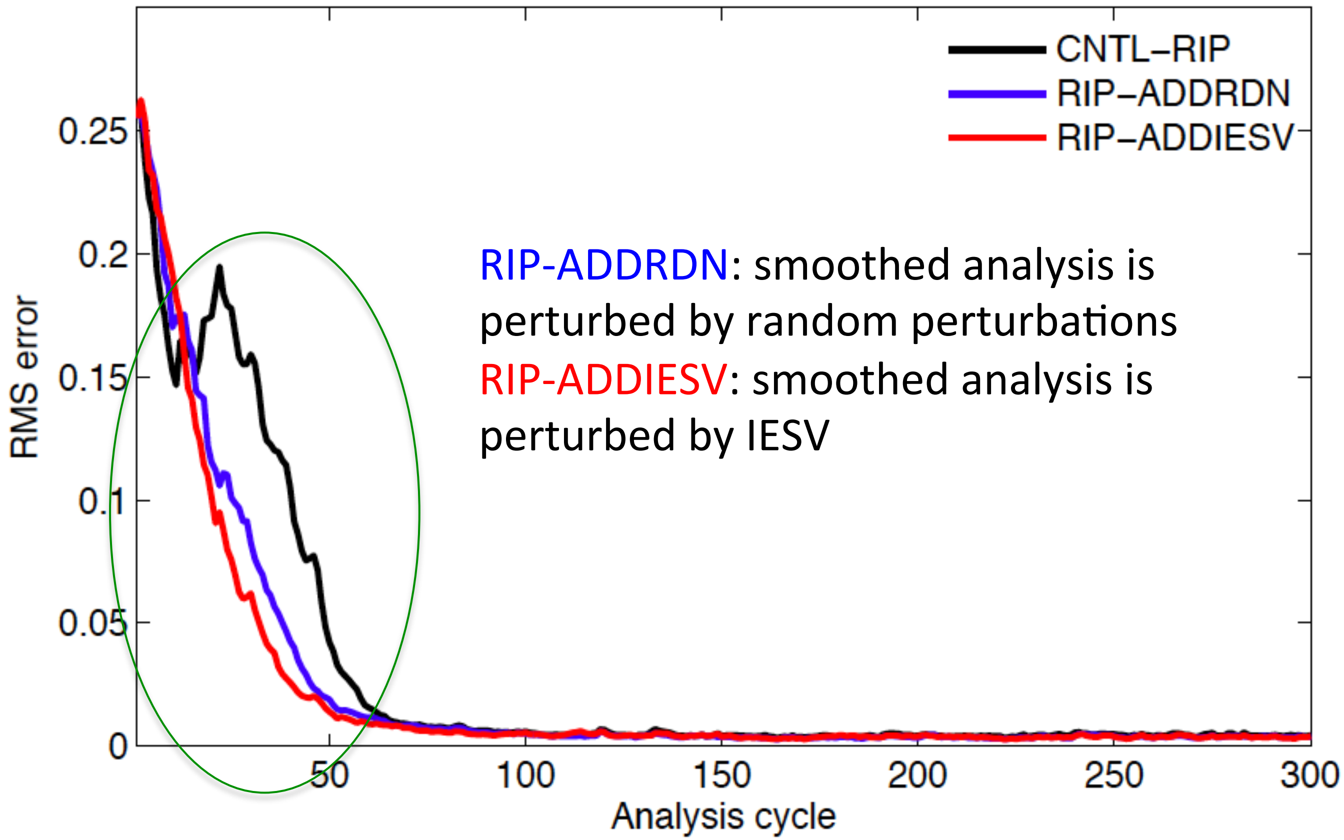
Apply IESV in LETKF-RIP

- The Running In Place (RIP) method (Kalnay and Yang, 2010) aims to accelerate EnKF's spin-up
  - The RIP method is designed to re-evolve the whole ensemble to catch up the true dynamics, represented by observations.
  - Both the accuracy of the mean state and structure of the ensemble-based covariance are improved.
  - RIP: no-cost smoother + iteration procedure

Steps for performing RIP

1. Apply the no-cost smoother at the previous analysis time
2. Perturb and re-evolve the smoothed analysis ensemble
3. With the improved background ensemble, re-assimilate observations at the current analysis time.
4. Steps 1-3 are repeated until the differences of the RMS innovation between two iterations is smaller than 5%.

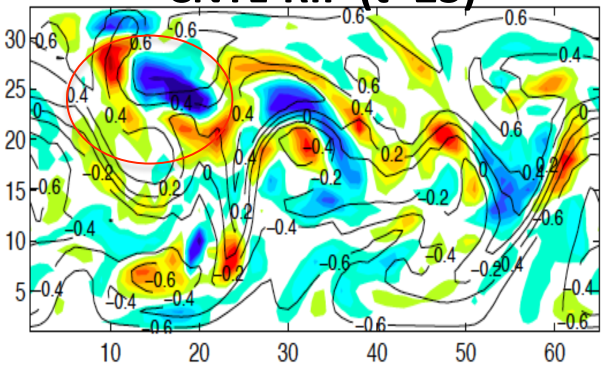
Apply IESV in LETKF-RIP



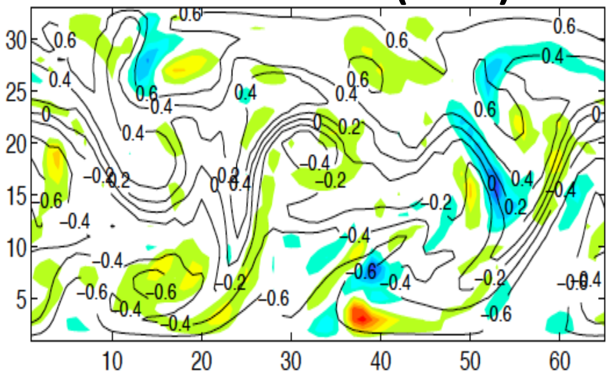
IESVs point out the fast growing directions of the errors and can further accelerate the spin-up of LETKF.

Correcting fast growing errors with LETKF-RIP

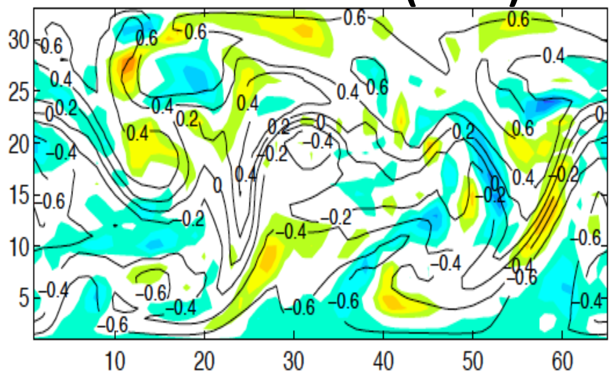
CNTL-RIP (t=23)



ADDIESV-RIP (t=23)



ADDRDN-RIP (t=23)



Large errors can be quickly removed in the ADDIESV-RIP analysis!

Summary

- ESVs representing the fast growing errors can be derived without the use of tangent linear/adjoint model.
  - Constructing ESVs with a EnKF framework is almost cost-free.
- ESVs better correlated with the background errors than the ensemble perturbations.
  - When ensemble perturbations are still developing the flow-dependent structure, ESVs are able to capture the fast growing modes already.
- The initial ESVs can be used for additive covariance inflation for EnKF.
  - The positive impact is particularly identified for areas with large errors.
  - When initial ESVs are applied in the RIP-iteration, the LETKF's spin-up period can be further shortened.