Estimating Observation Impact in a Hybrid Data Assimilation System: Experiments with a Simple Model

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Overview

- Goal
- Sensitivity Theory
  - Adjoint Sensitivity Theory
  - Ensemble Sensitivity Theory
- Observation Impact
- Hybrid Data Assimilation
- Experiments
- Summary

Goal

Evaluate observation impact in a hybrid data assimilation system using ensemble statistics and validate them with adjoint methods

Sensitivity Theory

Definitions

- state vector, perturbation, ensemble matrix: $\mathbf{x}$, $\delta\mathbf{x}$, $\delta\mathbf{X}$
- forecast metric, perturbation, ensemble vector: J, δJ, $\delta\mathbf{J}$
- analysis time, verification time: t_0, t
- TLM: $\mathbf{M}_{t,t_0} = \mathbf{M}_{t,t-\delta t} \cdots \mathbf{M}_{t_i+\delta t,t_i} \cdots \mathbf{M}_{t_0+\delta t,t_0}$
- ADJ: $\mathbf{M}^{\mathrm{T}}_{t,t_0} = \mathbf{M}^{\mathrm{T}}_{t_0+\delta t,t_0} \cdots \mathbf{M}^{\mathrm{T}}_{t_i+\delta t,t_i} \cdots \mathbf{M}^{\mathrm{T}}_{t,t-\delta t}$

Adjoint Sensitivity Theory

Linearized dynamics and metric response:

$$\delta \mathbf{x}_t \approx \mathbf{M}_{t,t_0} \delta \mathbf{x}_{t_0}$$

$$\delta J \approx \frac{\partial J}{\partial \mathbf{x}_t}^{\mathrm{T}} \delta \mathbf{x}_t$$

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Ensemble Sensitivity Theory

δJ and δx are random variables, assume Gaussian distributed:

- Let $\{\cdot\}$ denote expectation.
- $\mathbf{P} = \left\{\delta\mathbf{x}_{t_0}\delta\mathbf{x}_{t_0}^{\mathrm{T}}\right\} \equiv cov\left(\mathbf{X}_{t_0}, \mathbf{X}_{t_0}\right)$ denote error covariance matrix

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Ancell and Hakim 2007

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$$cov\left(\mathbf{J}, \mathbf{X}_{t_0}\right) = \frac{\partial J}{\partial \mathbf{x}_{t_0}}^{\mathrm{T}} \mathbf{P}$$

Ancell and Hakim 2007

Adjoint v/s Ensemble Sensitivity Theory

Ensemble method recovers adjoint sensitivity

- $\frac{\partial J}{\partial \mathbf{x}_{t_0}} = \mathbf{P}^{-1} cov\left(\mathbf{X}_{t_0}, \mathbf{J}\right) = \mathbf{M}^{\mathrm{T}}_{t,t_0} \frac{\partial J}{\partial \mathbf{x}_t}$
- Simultaneous multivariate regression

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Ensemble Sensitivity - Pros and Cons

Pros

- No adjoint model is required.
- No assumptions about on/off or moist processes.
- Rapidly evaluate many J (cf. new adjoint run for each J).
- Can apply statistical significance testing.

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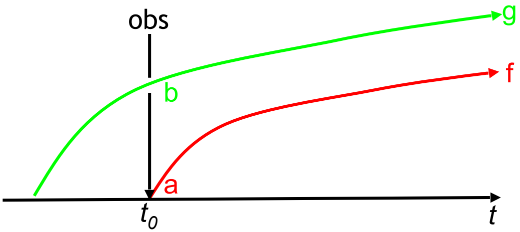
- Rapidly evaluate many J (cf. new adjoint run for each J).
- Can apply statistical significance testing.

Cons

- Sampling error.
- Computing $\mathbf{P}^{-1}$ is impractical for high-dimensional problems.

Observation Impact



$$\delta e_f^g = \left\langle (\mathbf{y} - \mathbf{H}\mathbf{x}_b)\,, \mathbf{K}^{\mathrm{T}} \left(\frac{\partial J_g}{\partial \mathbf{x}_b} + \frac{\partial J_f}{\partial \mathbf{x}_a} \right) \right\rangle$$

Langland and Baker 2004

Observation Impact

Adjoint Framework

$$\mathbf{K}^{\mathrm{T}}\frac{\partial J_g}{\partial \mathbf{x}_b} = \mathbf{K}^{\mathrm{T}}\mathbf{M}_{t,t_0}^{\mathrm{T}}\frac{\partial J_g}{\partial \mathbf{x}_g}$$

$$\mathbf{K}^{\mathrm{T}}\frac{\partial J_f}{\partial \mathbf{x}_a} = \mathbf{K}^{\mathrm{T}}\mathbf{M}_{t,t_0}^{\mathrm{T}}\frac{\partial J_f}{\partial \mathbf{x}_f}$$

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$$\mathbf{K} = \mathbf{B}\mathbf{H}^{\mathrm{T}} \left[\mathbf{H}\mathbf{B}\mathbf{H}^{\mathrm{T}} + \mathbf{R}\right]^{-1} = \mathbf{A}\mathbf{H}^{\mathrm{T}}\mathbf{R}^{-1}$$

Observation Impact

Adjoint Framework

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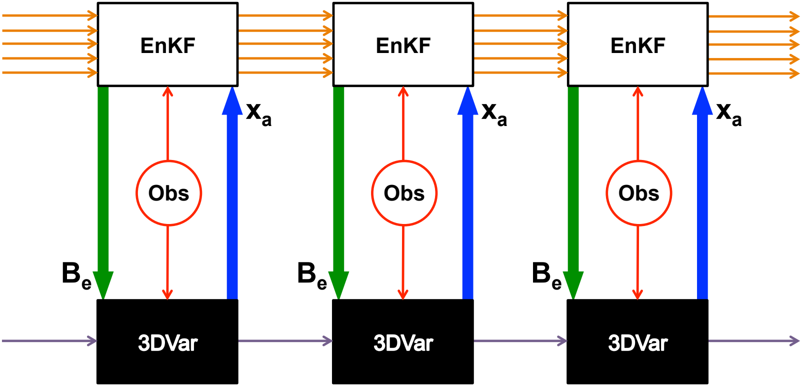
Ensemble Framework

$$\mathbf{K}^{\mathrm{T}} \frac{\partial J_g}{\partial \mathbf{x}_b} = \mathbf{K}^{\mathrm{T}} \mathbf{B}^{-1} cov\left(\mathbf{X}_b, \mathbf{J}_g\right) = \left[\mathbf{H}\mathbf{B}\mathbf{H}^{\mathrm{T}} + \mathbf{R}\right]^{-1} cov\left(\mathbf{H}\mathbf{X}_b, \mathbf{J}_g\right)$$

$$\mathbf{K}^{\mathrm{T}} \frac{\partial J_f}{\partial \mathbf{x}_a} = \mathbf{K}^{\mathrm{T}} \mathbf{A}^{-1} cov\left(\mathbf{X}_a, \mathbf{J}_f\right) = \mathbf{R}^{-1} cov\left(\mathbf{H}\mathbf{X}_a, \mathbf{J}_f\right)$$

Hybrid Data Assimilation System



$$C(x) = \frac{1}{2}\left[\mathbf{x} - \mathbf{x}_b\right]^{\mathrm{T}} \mathbf{B}_h^{-1} \left[\mathbf{x} - \mathbf{x}_b\right] + \frac{1}{2}\left[\mathbf{H}\mathbf{x}_b - \mathbf{y}\right]^{\mathrm{T}} \mathbf{R}^{-1} \left[\mathbf{H}\mathbf{x}_b - \mathbf{y}\right]$$

$$\mathbf{B}_h = (1-\beta)\,\mathbf{B}_s + \beta\mathbf{B}_e \circ \mathbf{L}$$

Model and DA Configuration

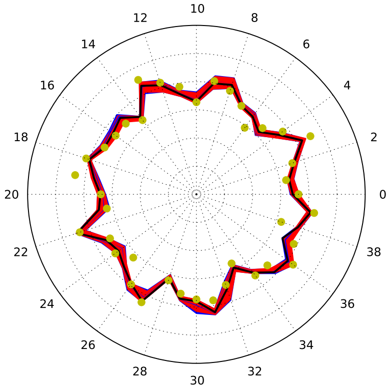
Lorenz 1996

$$\frac{\partial X_i}{\partial t} = (X_{i+1} - X_{i-2})\, X_{i-1} - X_i + F$$

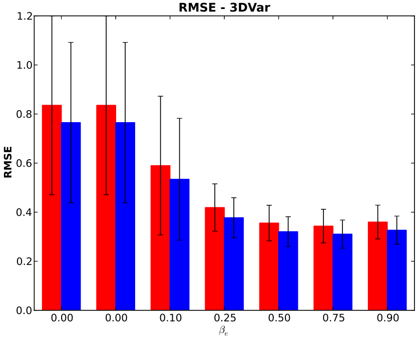
I = 40
F = 8.0 (perfect model), 8.4

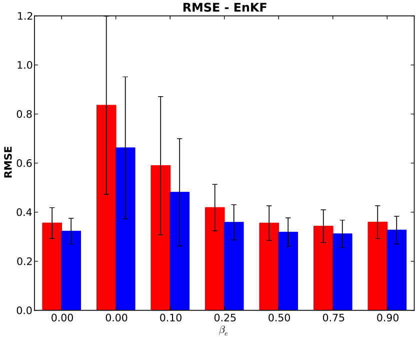
Data Assimilation

Three-D Var
$\mathbf{B}_s$ derived from very long integration with EnKF
EnSRF, Ne = 20
Inflation = 2%
Localization = 4 points
$\beta_s = 0.25$, $\beta_e = 0.75$



Tuning the hybrid data assimilation system





Prior RMSE
Posterior RMSE

Observation Impact Experiments

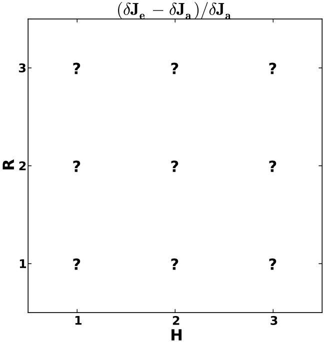
Varying Observation Location

- H = 1 : All 40 observed
- H = 2 : Alternate 20 observed
- H = 3 : Random 20 observed

Varying Observation Quality

- R = 1 : Uniform ob. quality
- R = 2 : Alternate good/bad ob.
- R = 3 : Random ob. quality

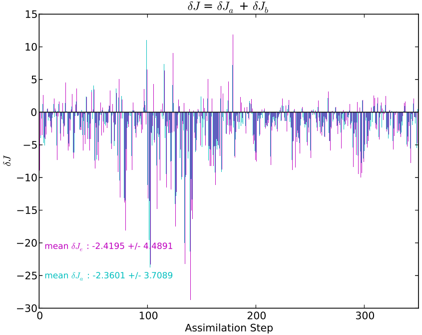
$$\text{Forecast error metric} = \mathbf{J} = \text{Total Energy} = \sum_{i}^{I} (x_f - x_v)^2$$

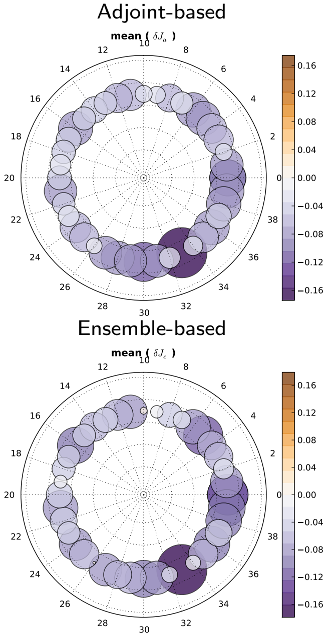


Experiment H = 1, R = 1

All 40 obs., Same quality

- assimilate obs. at all locations
- all obs. have same obs. error

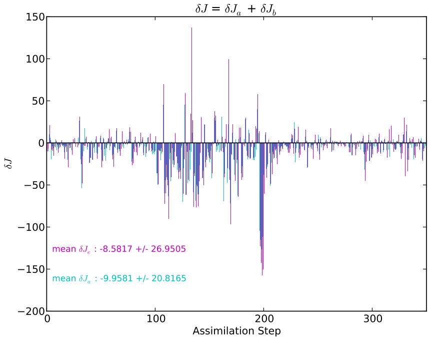


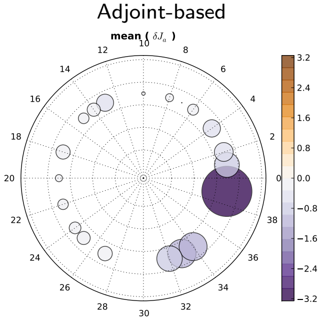


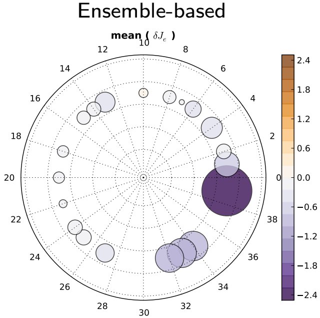
Experiment H = 3, R = 3

Random 20 obs., Random quality

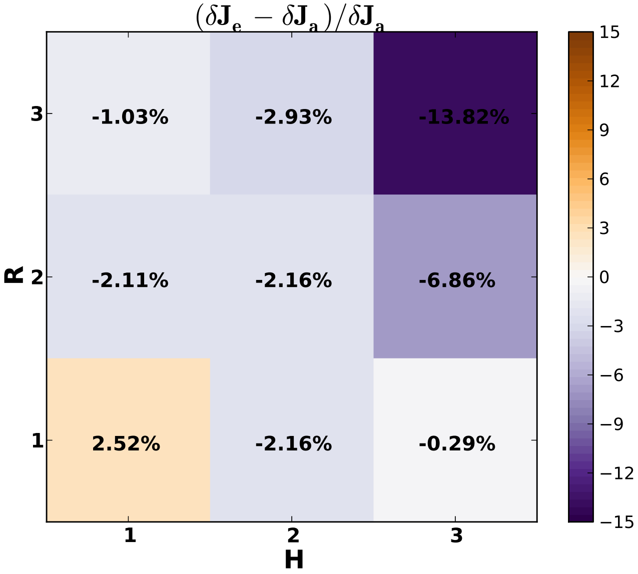
- assimilate obs. at all locations
- all obs. have same obs. error







Experiments - Result Summary



Summary and Future Work

Summary

- Ensemble-method recovers adjoint sensitivity
- Applied ensemble sensitivity and validated against adjoint method based observation impact in Lorenz 1996.

Future Work

- Explore ensemble technique in a full NWP system.
- Extend observation impact to 4DEnsVar.
- Explore localization in observation impact calculations