An Adaptive Covariance Estimation Method

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Outline

- Mathematical formulation
- Belanger's method
- A modified version of Belanger's method
- Numerical results
- Future work

Mathematical formulation

- True model:

$$X_{n+1} = F_n X_n + w_n$$

Deterministic model operator

State variable

System noise (random variable)

Mathematical formulation

- Observational model:

$$y_n = H_n X_n + \xi_n$$

- y_n → observation
- H_n → Observation operator
- ξ_n → Observation noise

Mathematical formulation

- Goal: find out the variance of w_n and ξ_n, which are denoted by Q and R.
- This is different from the model error problem.

Belanger's method

- Construct a new set of “observations” for Q and R from the existing observations:

$$\mathcal{Y}_{n,l} = y_n y_{n-l}^\top$$

Observation at time t_n

Observation at time t_{n-l}

Belanger's method

This newly constructed observations satisfy:

- 1, $\mathbb{E}[\mathcal{Y}_{n,l}] = \mathcal{H}_{n,l}(Q, R)$

  linear

- 2, $Var(\mathcal{Y}_{n,l})$ can be computed recursively.

Belanger's method

Process:

1, primary filter (Kalman filter)

2, secondary filter:

implement Kalman filter on the observation model:

$$\mathcal{Y}_{n,l} = \mathcal{H}_{n,l}(Q,R) + \eta_{n,l}$$

to estimate Q and R.

A modified version of Belanger's method

- Starting from the relation:

$$\mathcal{Y}_{n,l} = \mathcal{H}_{n,l}(Q,R) + \eta_{n,l}$$

- consider

$$\frac{1}{n-l}\sum_{k=l+1}^{n}\mathcal{Y}_{n,l} = \frac{1}{n-l}\left(\sum_{k=l+1}^{n}\mathcal{H}_{n,l}\right)(Q,R) + \frac{1}{n-l}\sum_{k=l+1}^{n}\eta_{n,l}$$

- and solve it directly using least-square method.

A modified version of Belanger's method

- Without computing the variance of the newly constructed observations, it is computationally cheaper than the original Belanger's method.

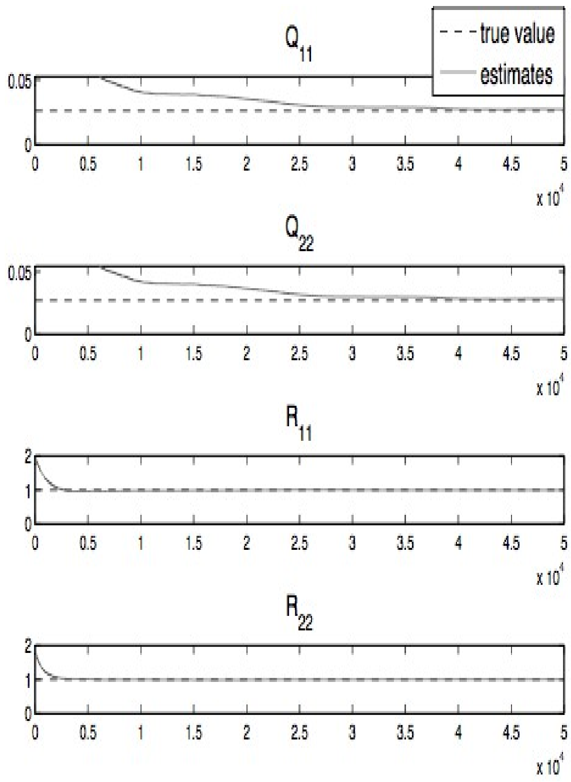
Numerical results

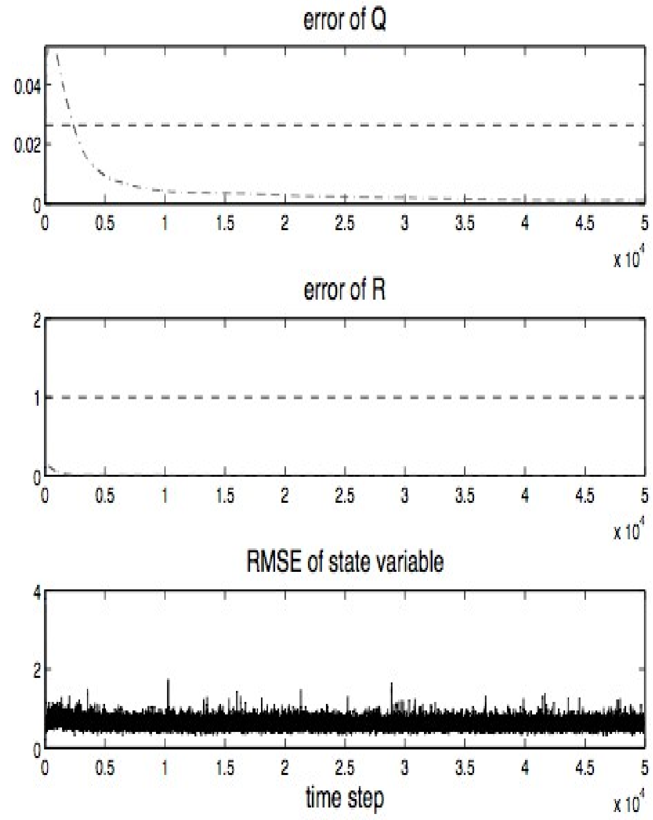
- Results with Lorenz96 model:

$$\frac{dx_i}{dt} = -x_{i-2}x_{i-1} + x_{i-1}x_{i+1} - x_i + F + \dot{W}_i$$

- 20 observation, R=I, Q=0.5dt

Numerical results





Future work

- First Implement this with Sequential Kalman filter/LETKF.