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An adaptive covariance relaxation method for ensemble data assimilation

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Outline

- Introduction: covariance inflation and innovation statistics
- Methods: to adaptively determine the inflation parameter
- Intercomparison of different methods using Lorenz-96 model
- Summary and further application challenges

The ensemble Kalman filter (EnKF)

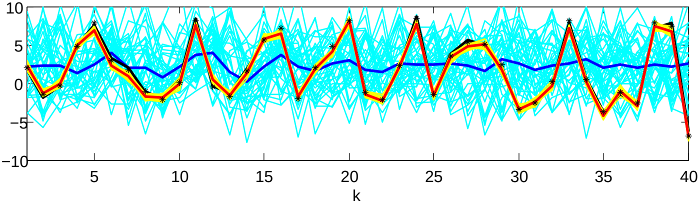
For observations j=1,2,...,p,
For members i=1,2,...,m,

$$\mathbf{x}_i \leftarrow \mathbf{x}_i + \mathbf{K}[y_j^o - h_j(\mathbf{x}_i)]$$

The Kalman gain:

$$\mathbf{K} = \frac{\text{cov}[\mathbf{x}, h_j(\mathbf{x})]}{\text{var}[h_j(\mathbf{x})] + \text{var}(y_j^o)}$$

In matrix format: $\mathbf{K} = \mathbf{PH}^T(\mathbf{HPH}^T + \mathbf{R})^{-1}$



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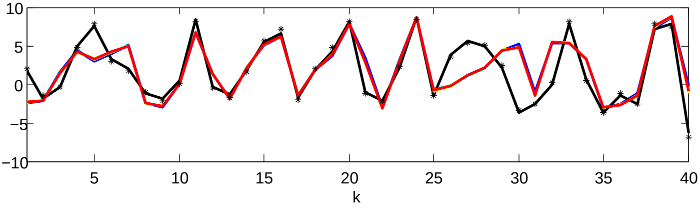
Filter divergence

When there are unrepresented model/sampling errors, the background error variance is underestimated. The observation is eventually ignored.

$$\mathbf{K} = \frac{\mathrm{cov}[\mathbf{x}, h_j(\mathbf{x})]}{\mathrm{var}[h_j(\mathbf{x})] + \mathrm{var}(y_j^o)} \sim 0$$



Solution: covariance inflation.

Covariance inflation

- multiplicative: $x \leftarrow \lambda x$ (Anderson and Anderson 1999)
- additive: $x \leftarrow x + \epsilon_c$ (Mitchell and Houtekamer 2000)
- relax-to-prior-perturbation: $x^a \leftarrow (1-\alpha)x^a + \alpha x^f$ (Zhang et al. 2004)
- relax-to-prior-spread: $x^a \leftarrow x^a \left(\alpha \frac{\sigma^f - \sigma^a}{\sigma^a} + 1\right)$ (Whitaker and Hamill 2012)

It is costly to tune these empirical methods, need adaptive methods.

Innovation statistics

Desroziers et al. (2005) derive

$$E[(\mathbf{y}^o - \mathbf{Hx})(\mathbf{y}^o - \mathbf{Hx})^T] = \mathbf{HPH}^T + \mathbf{R}$$

Consistency ratio $= \sqrt{RHS/LHS}$

We can estimate inflation factor λ so that

$$E[(\mathbf{y}^o - \mathbf{Hx})(\mathbf{y}^o - \mathbf{Hx})^T] = \lambda^2\mathbf{HPH}^T + \mathbf{R}$$

It requires a relatively large sample of $\mathbf{y}^o$ for this statistical relation to hold.

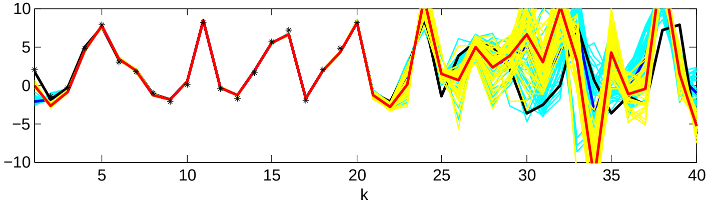
Temporally varying inflation

Issue: small innovation sample size at each assimilation cycle.

- use temporally smoothed λ as domain-wise inflation (Li et al. 2009; Miyoshi 2011)
  introduces a tunable smoothing parameter
- treat innovation $y^o - h(\mathbf{x})$ as a random variable and determines λ with Bayesian filter. (Anderson 2009)
  introduces a tunable parameter: σ_λ^2

Spatially varying inflation

The innovation statistics is performed in observation space. So λ only works for observed area.



- Anderson (2009) treats λ separately for each location, assuming $\mathrm{corr}(\lambda_k, \lambda) = \mathrm{corr}(x_k, h(\mathbf{x}))$
- The relax-to-prior-spread method uses a spatial mask: $(\sigma^f - \sigma^a)/\sigma^a$.

Adaptive covariance relaxation (ACR)

We propose an ACR method that combines advantages of the existing methods.

Use spatial mask from RTPS and determine a temporally smoothed α with innovation statistics:

$$(1+\gamma)\left(\alpha\frac{\bar{\sigma}_y^f - \bar{\sigma}_y^a}{\bar{\sigma}_y^a} + 1\right) = \lambda$$

Assume that posterior spread after relaxation will grow by a factor of $(1+\gamma)$ after time integration. We use $\gamma = 0$ because our cycling period is short.

Temporal smoothing: $\alpha_t \leftarrow \alpha_{t-1} + (\alpha_t - \alpha_{t-1})/\tau$

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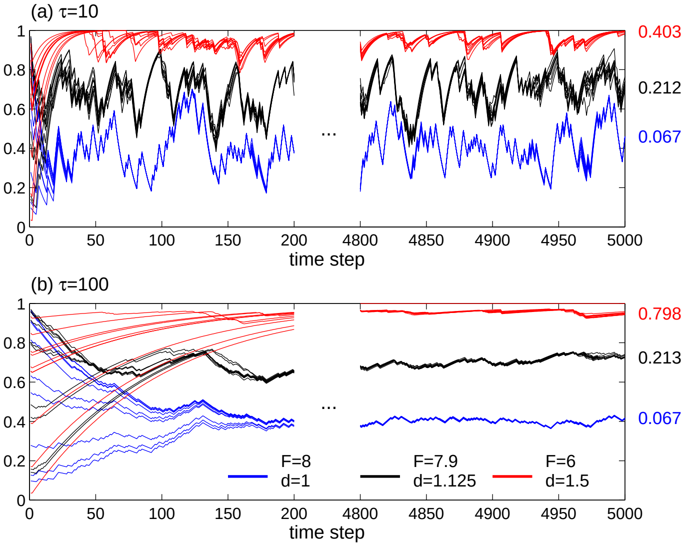
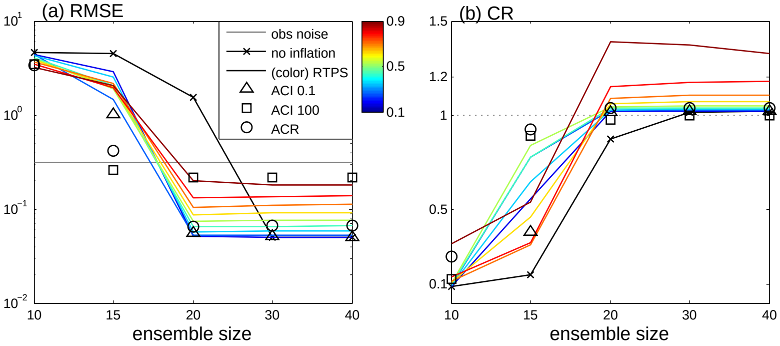
Time evolution of α



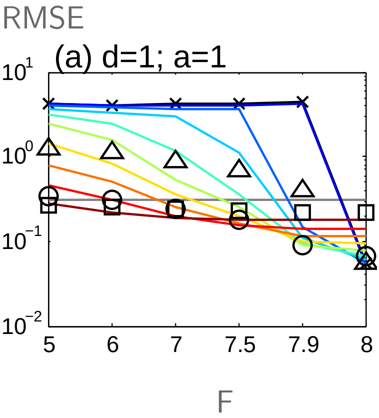
Performance with presence of sampling error

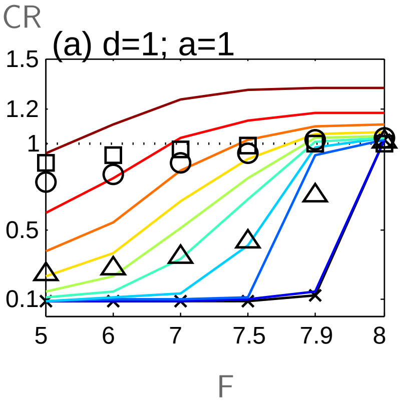


ACI 0.1 means Anderson 2009 method with $\sigma_\lambda^2 = 0.1$. The larger σ_λ^2, the more fit to each innovation.

Performance with presence of model error

$$\frac{dx_k}{dt} = ax_{k-1}x_{k-2} - x_{k-1}x_{k+1} - dx_k + F$$





obs noise — no inflation — (color) RTPS △ ACI 0.1 □ ACI 100 ○ ACR

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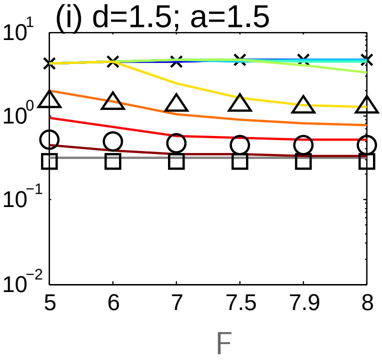
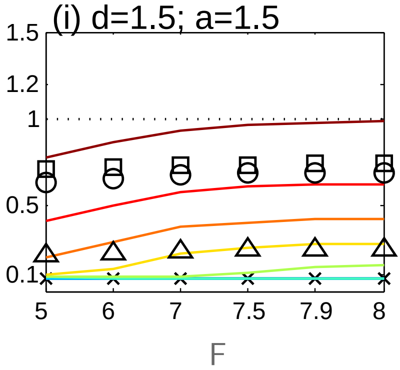
Performance with presence of model error

$$\frac{dx_k}{dt} = ax_{k-1}x_{k-2} - x_{k-1}x_{k+1} - dx_k + F$$

RMSE



CR



obs noise — no inflation — (color) RTPS — △ ACI 0.1 — □ ACI 100 — ○ ACR

Summary and future application

- The ACR method calculates α in RTPS adaptively. Results show that it ensures filter performance.
- temporal smoothing is used to achieve more robust innovation statistics.
- challenges for real atmospheric model application: nonlinear $h()$, nonzero γ and changing observation network.