Test of Adaptive Covariance Inflation Methods on the Lorenz-96 model

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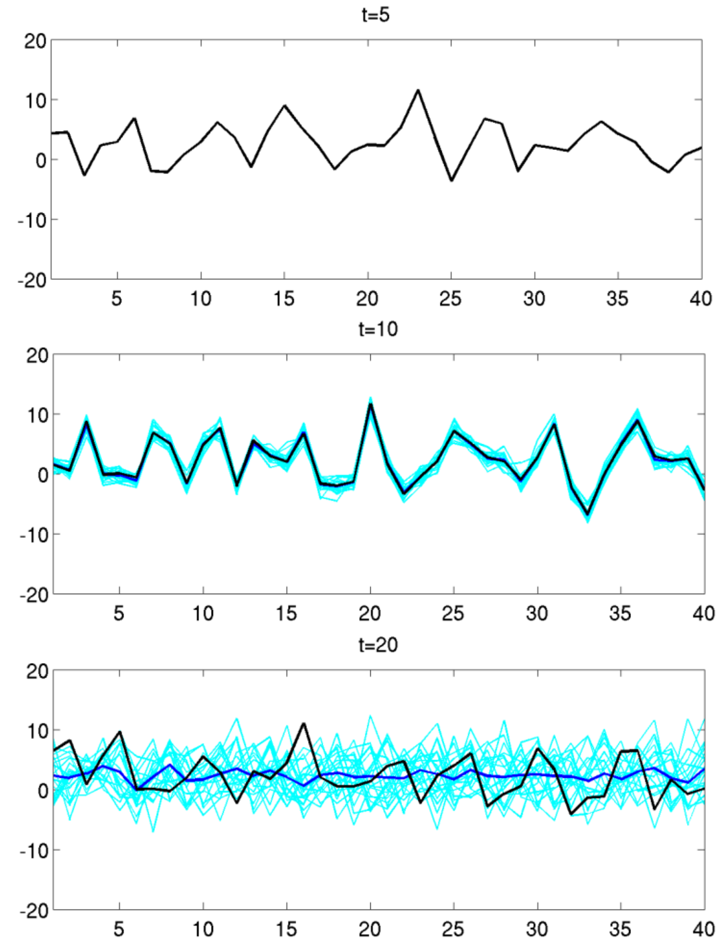
Dec 12, 2013

Lorenz-96 model

$dx_i/dt = x_{i+1}\, x_{i-1} - x_{i-2}\, x_{i-1} - x_i + F$
for cyclic $i=1,\ldots,K$

K=40
Δt=0.05
F=8
40 ensemble members



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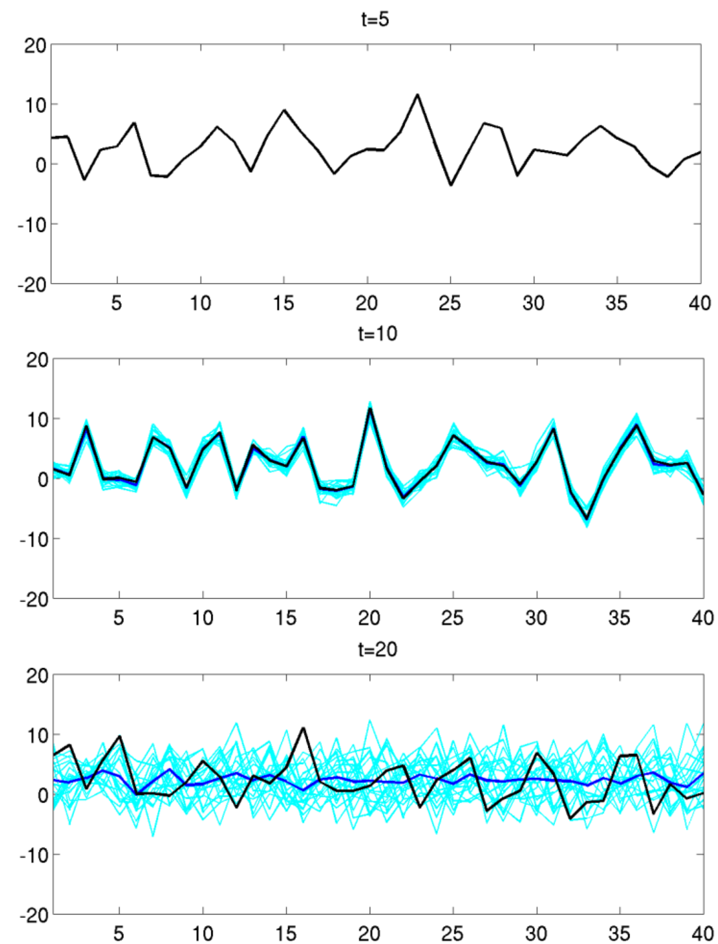
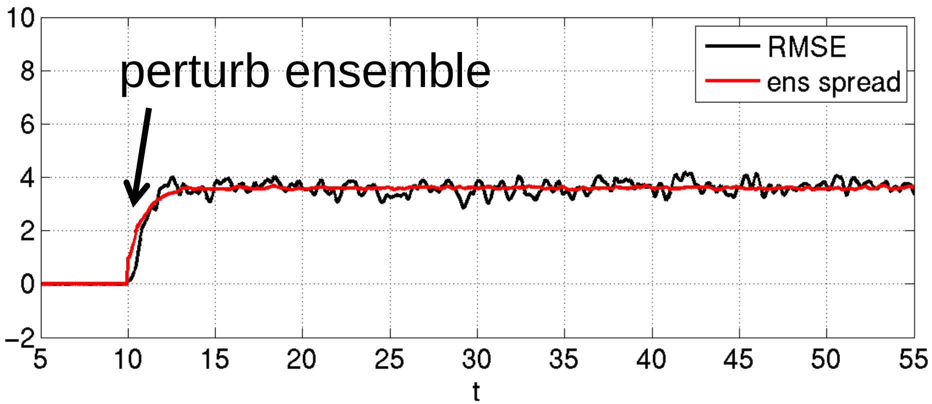
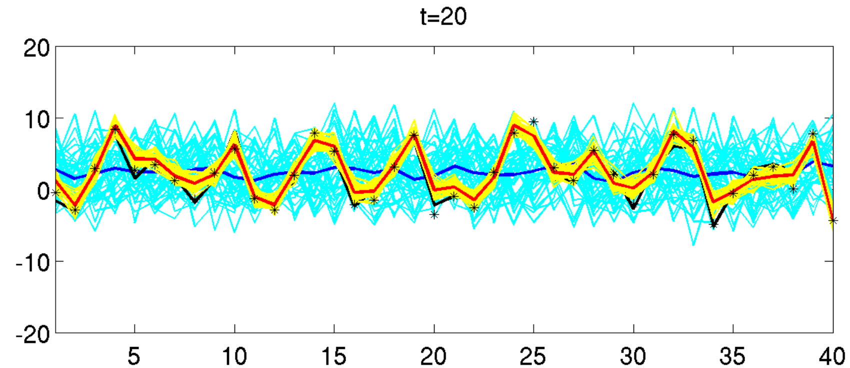


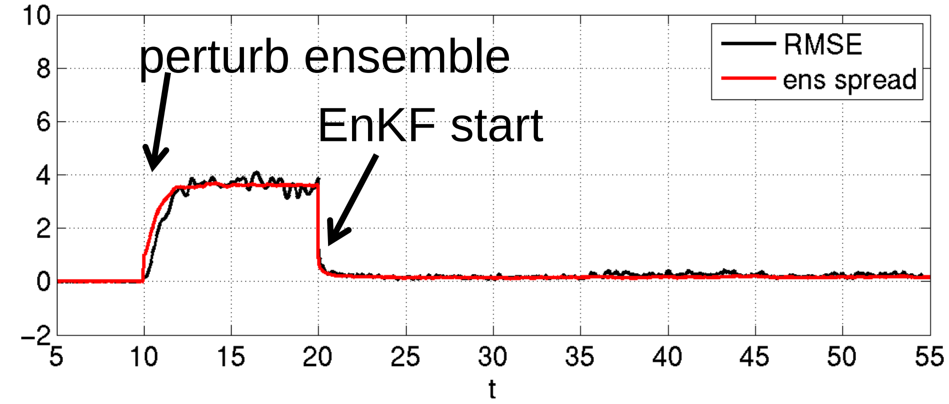
Error statistics:
RMSE = $\mathrm{sqrt}(\Sigma_i\, (\overline{\mathbf{x}}_i-\mathbf{x}^t_i)^2 / K)$
ens spread = $\Sigma_i\, std(\mathbf{x}_i) / K$



EnKF

40 ensemble members
observations located on each grid points (H=1)
random observational error ~ N(0,1)
assimilate at each time step





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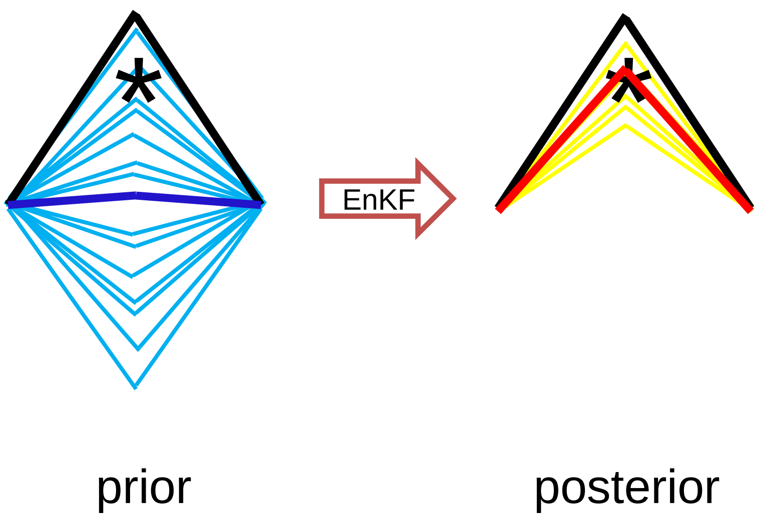
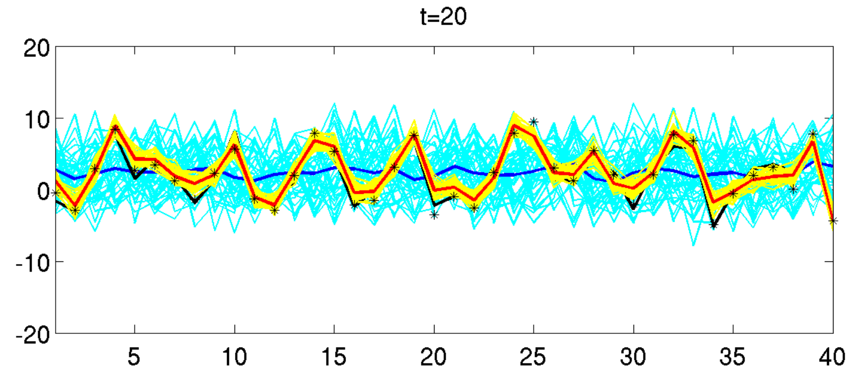
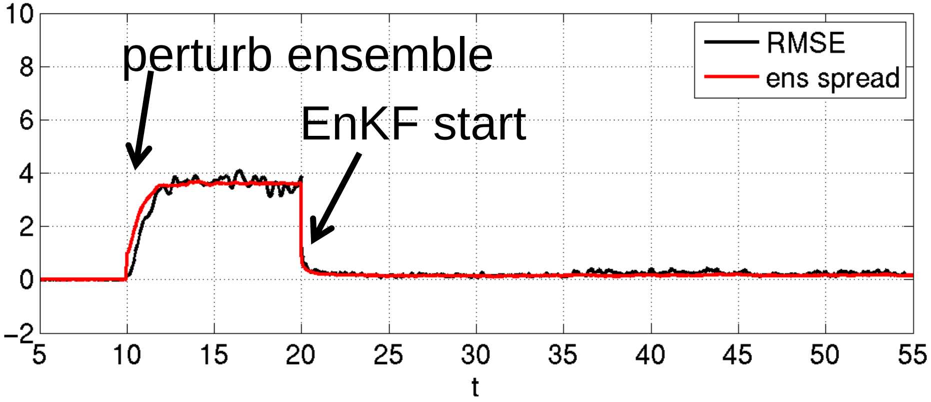


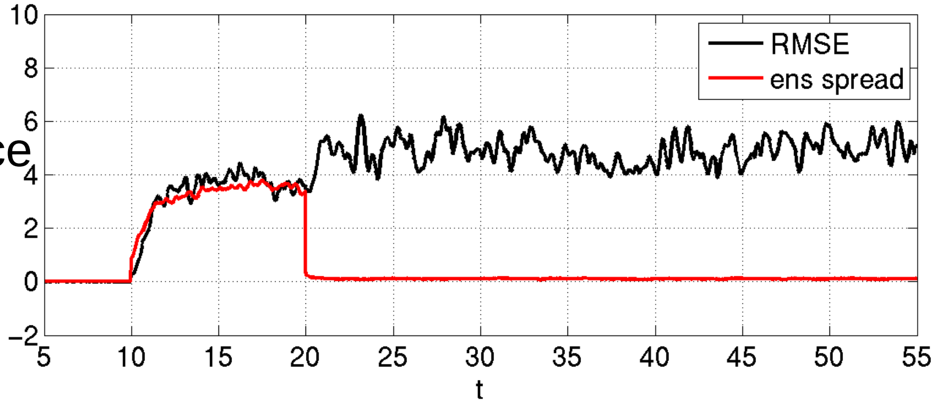




Sampling error

6 ensemble members

not sufficient to resolve covariance structure:
erroneous spatial correlation
filter divergence



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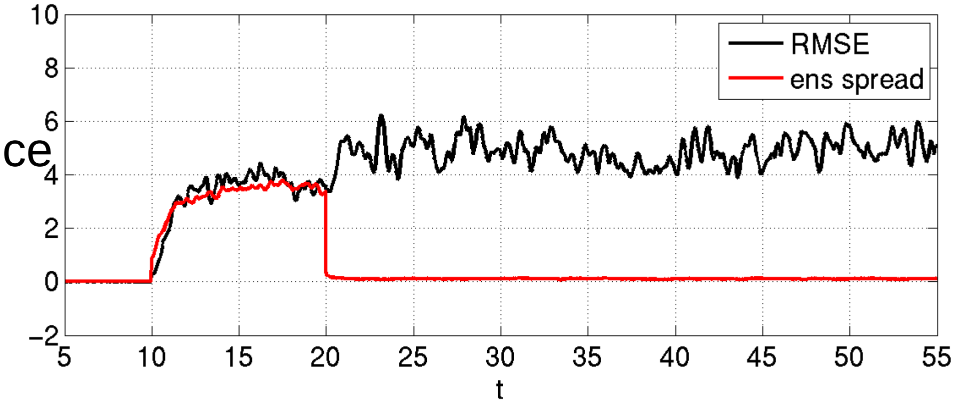
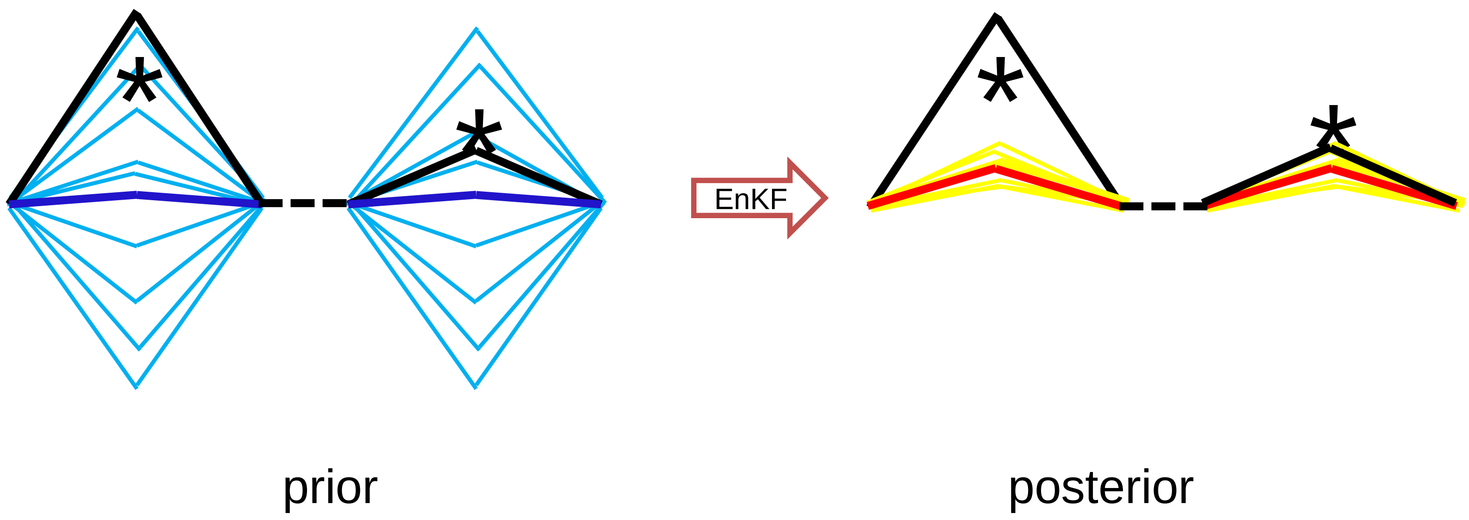
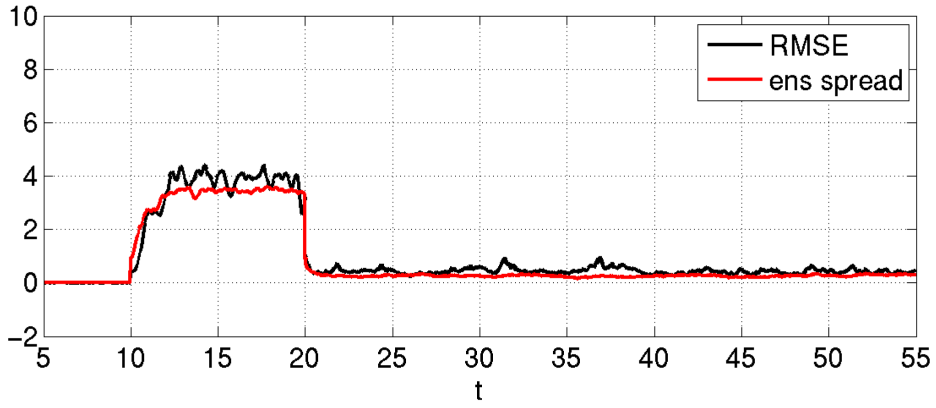




Sampling error

Localization remove spurious correlation
(Mitchell and Houtekamer 2000)
Gaspari-Cohn taper function
half width=1



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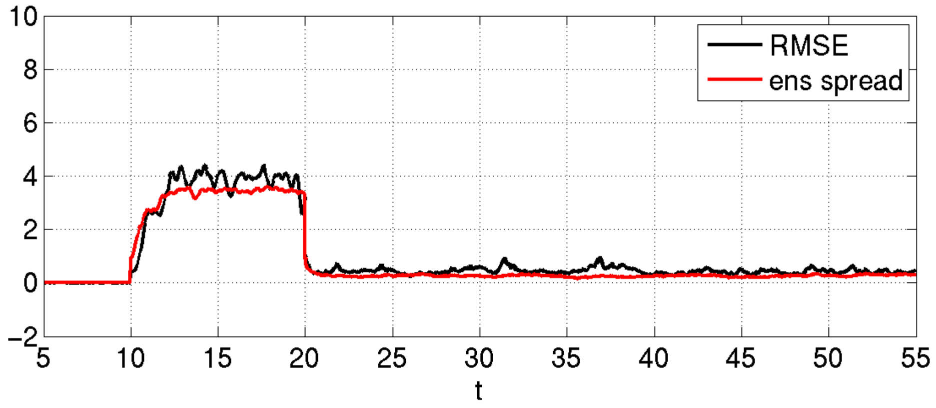
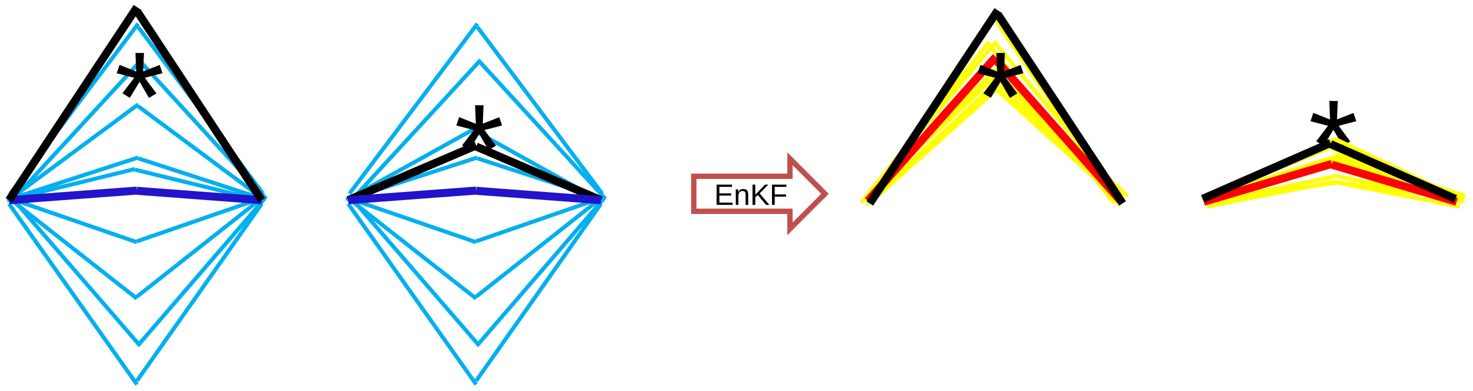




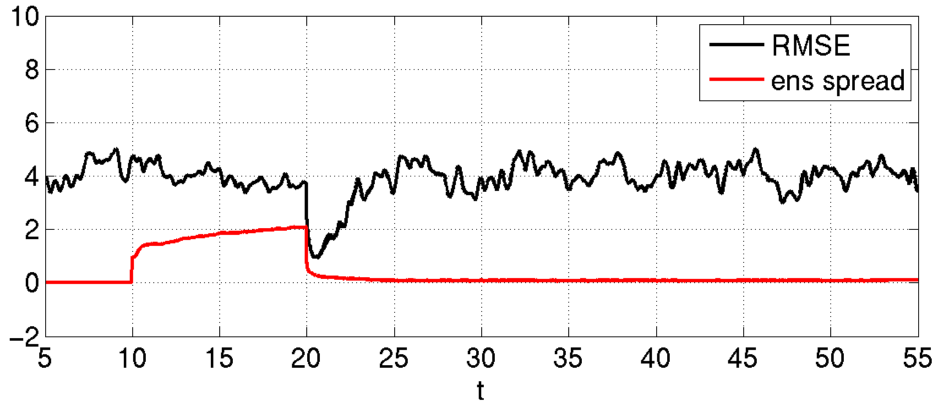
prior

posterior

Model error

40 ensemble members

true model F=8
ensemble members use
wrong model F=5



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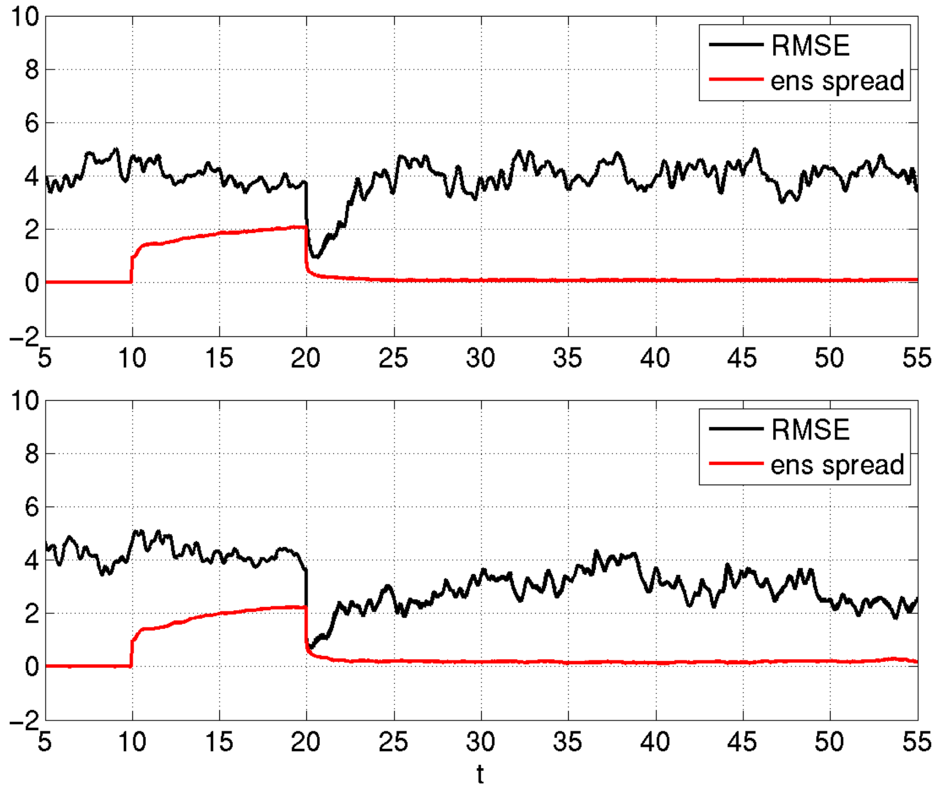
=> not helping!



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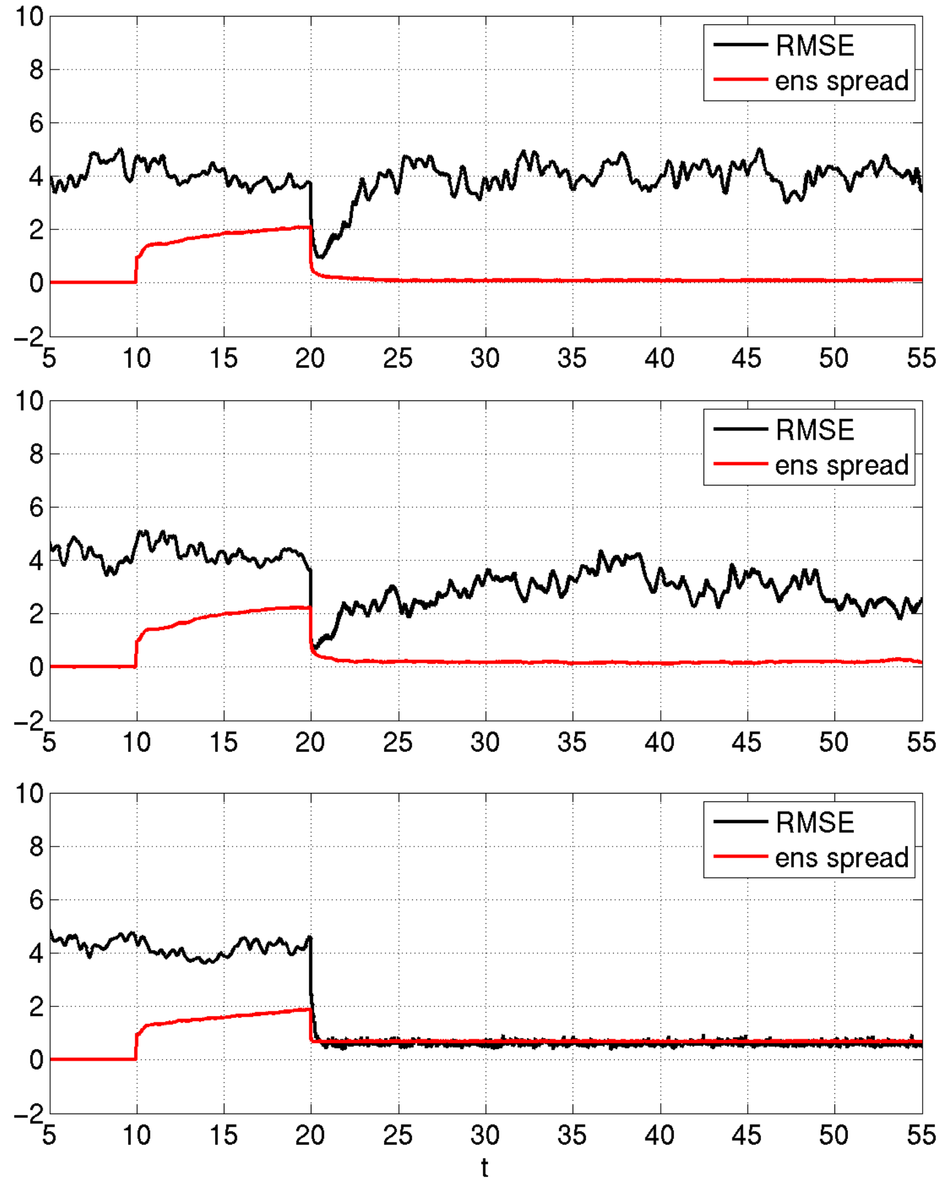
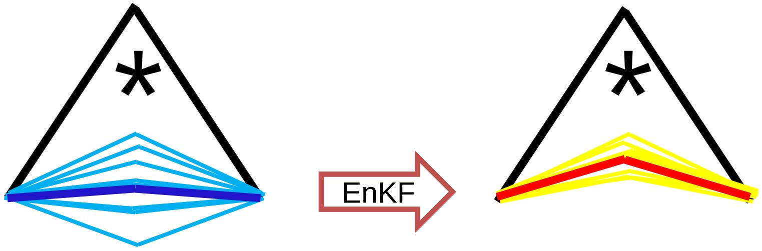
use localization, half width=1
=> not helping!

use covariance inflation λ^2=1.5
=> solved!



Covariance inflation

model error: unknown to EnKF
prior spread too small (too confident)



prior posterior

Covariance inflation

model error: unknown to EnKF
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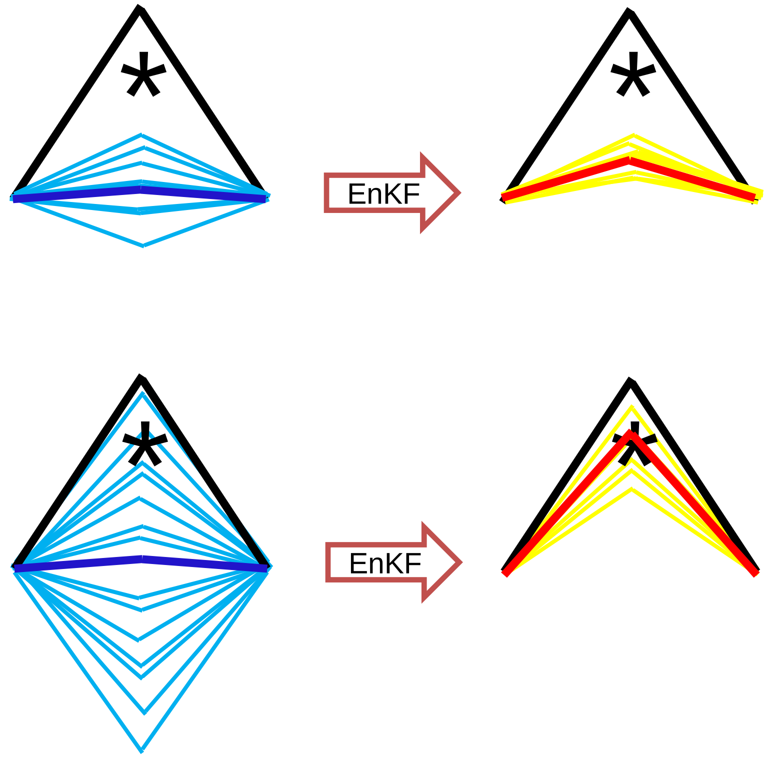
covariance inflation (Anderson and Anderson 1999)

$$x^b \leftarrow \lambda\, x^b$$

$$\sigma^b \leftarrow \lambda\, \sigma^b$$

x denote ensemble perturbations
$\sigma^2 = \Sigma\, x^2 / (N\text{-}1)$

goal: $\mathrm{RMSE}^2 = \sigma^{b\,2} + \sigma^{o\,2}$

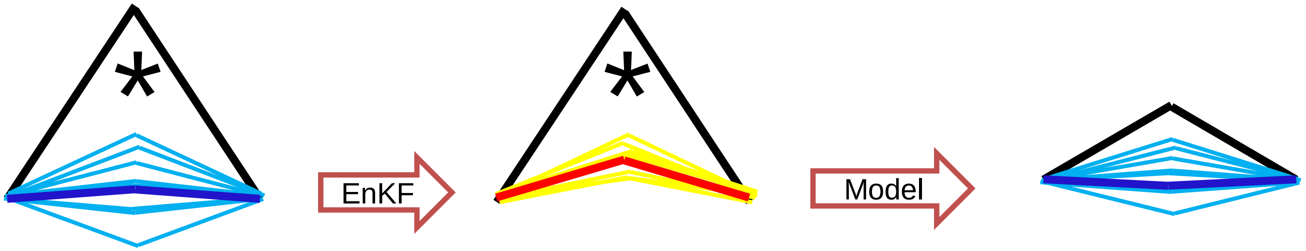


prior posterior

Relaxation methods

relaxation to prior perturbation (RTPP)
(Zhang et al. 2004)

$$x^a \leftarrow (1\text{-}\alpha)\, {\color{red}x^a} + \alpha\, {\color{blue}x^b}$$



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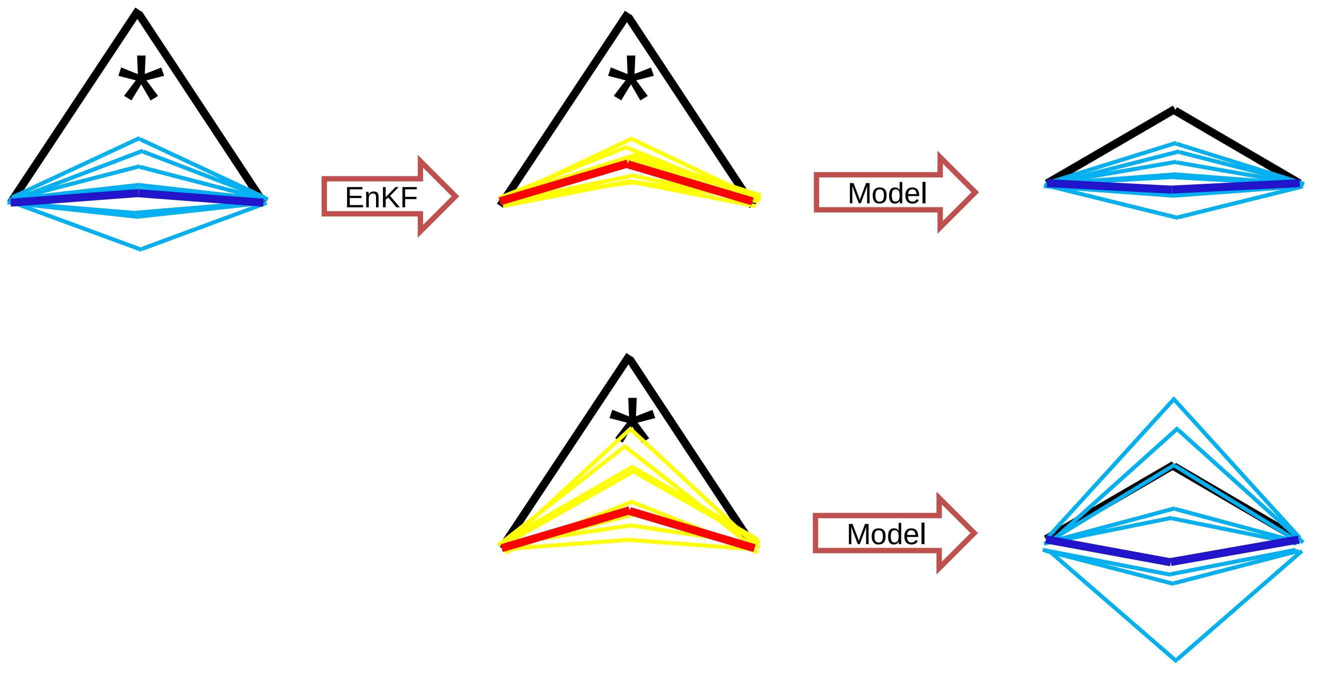

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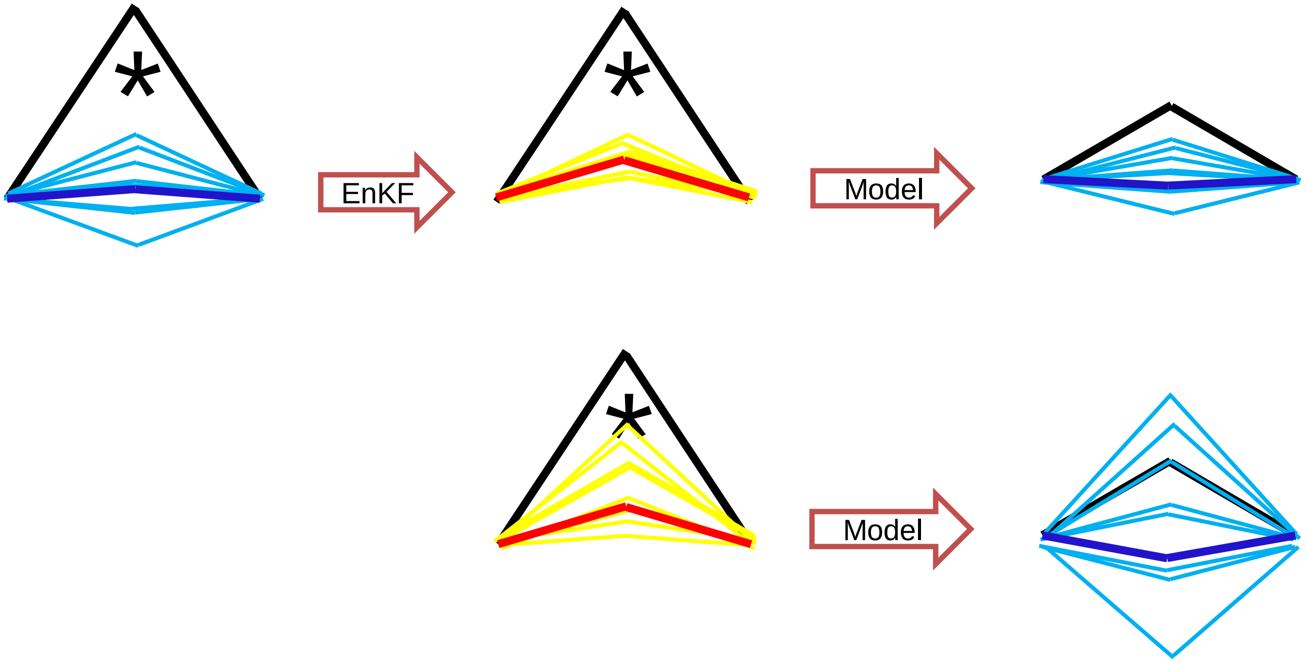
advantage: conserves some prior perturbation structure
disadvantage: takes effect later, rely on model to increase the spread



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relaxation to prior spread (RTPS)
(Whitaker and Hamill 2012)

$$\sigma^a \leftarrow (1-\alpha)\, \sigma^a + \alpha\, \sigma^b$$

$x^a \leftarrow \lambda\, x^a$, where $\lambda = \alpha(\sigma^b-\sigma^a)/\sigma^a+1$

Adaptive methods

It is costly to tune the inflation/relaxation methods (λ, α)

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adaptive inflation (Anderson 2007)

$$\mathrm{RMSE}^{b\,2} = \lambda^{o\,2}\,\sigma^{b\,2} + \sigma^{o\,2}$$

update λ, σ_λ with λ^o using Bayesian inference

spatially varying λ (Anderson 2008)

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update λ, σ_λ with λ^o using Bayesian inference

spatially varying λ (Anderson 2008)

adaptive relaxation (new)

$$\mathrm{RMSE}^{a\,2} = \lambda^{2}\,\sigma^{a\,2} + \sigma^{o\,2}$$

$$x^a \leftarrow (1\text{-}\alpha)\,x^a + \alpha\,x^b = \lambda\,x^a$$

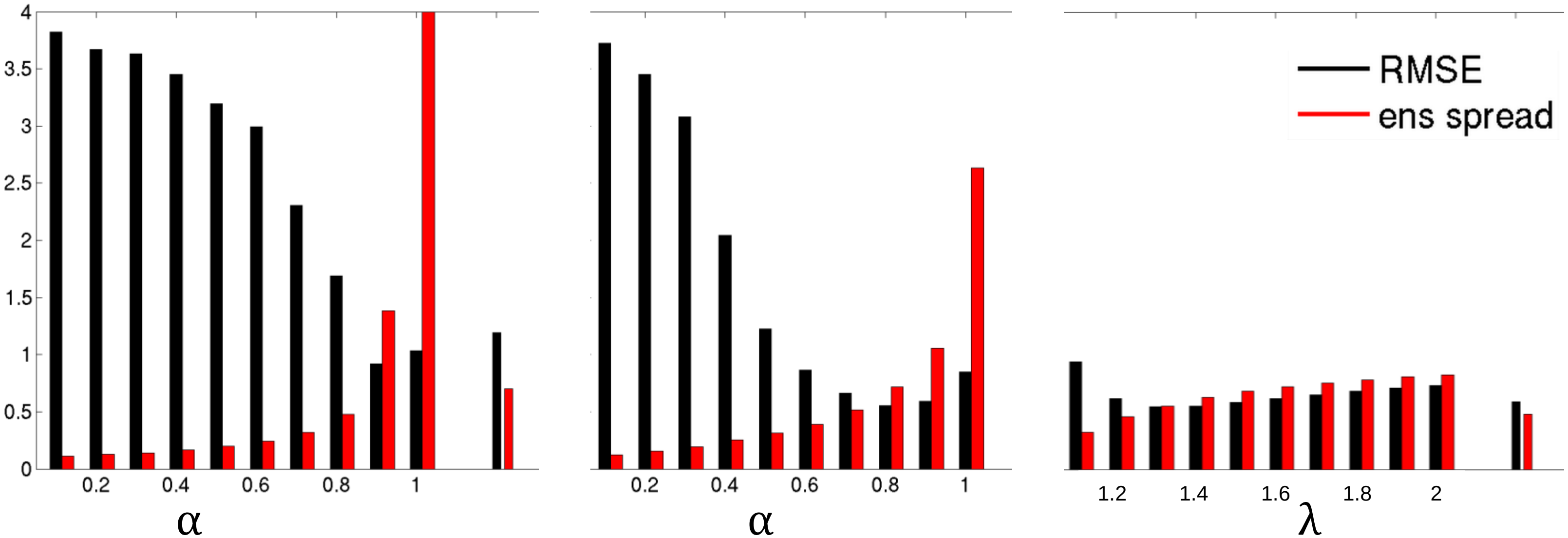
decrease in spread $x^b = (\sigma^b/\sigma^a)\,x^a = \gamma\,x^a$

$$(1\text{-}\alpha) + \alpha\,\gamma = \lambda$$

$$\alpha = (\lambda\text{-}1)/(\gamma\text{-}1)$$

Comparison among methods

40 members, F=5 model error



RTPP

adaptive RTPP

RTPS

Inflation

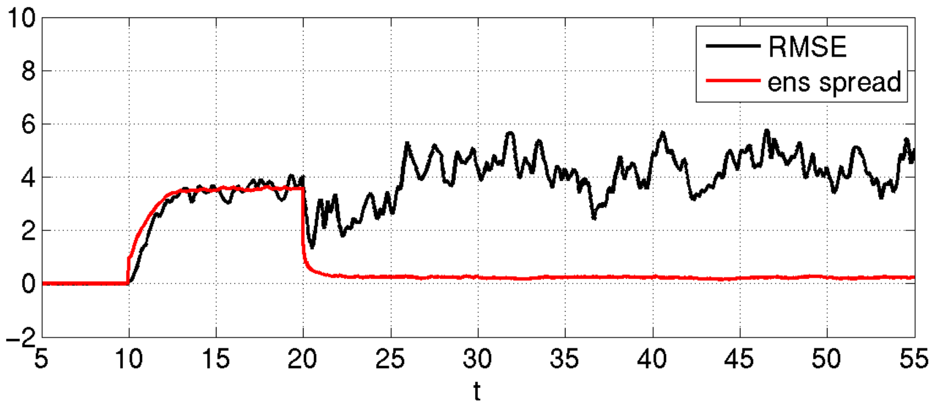
adaptive inflation

Other error sources

non-Gaussian error covariance

observations located off grid:
nonlinearity in the *true* H operator

observations located **randomly**:
partial coverage: too sparse / too dense ⟵ test inflation methods

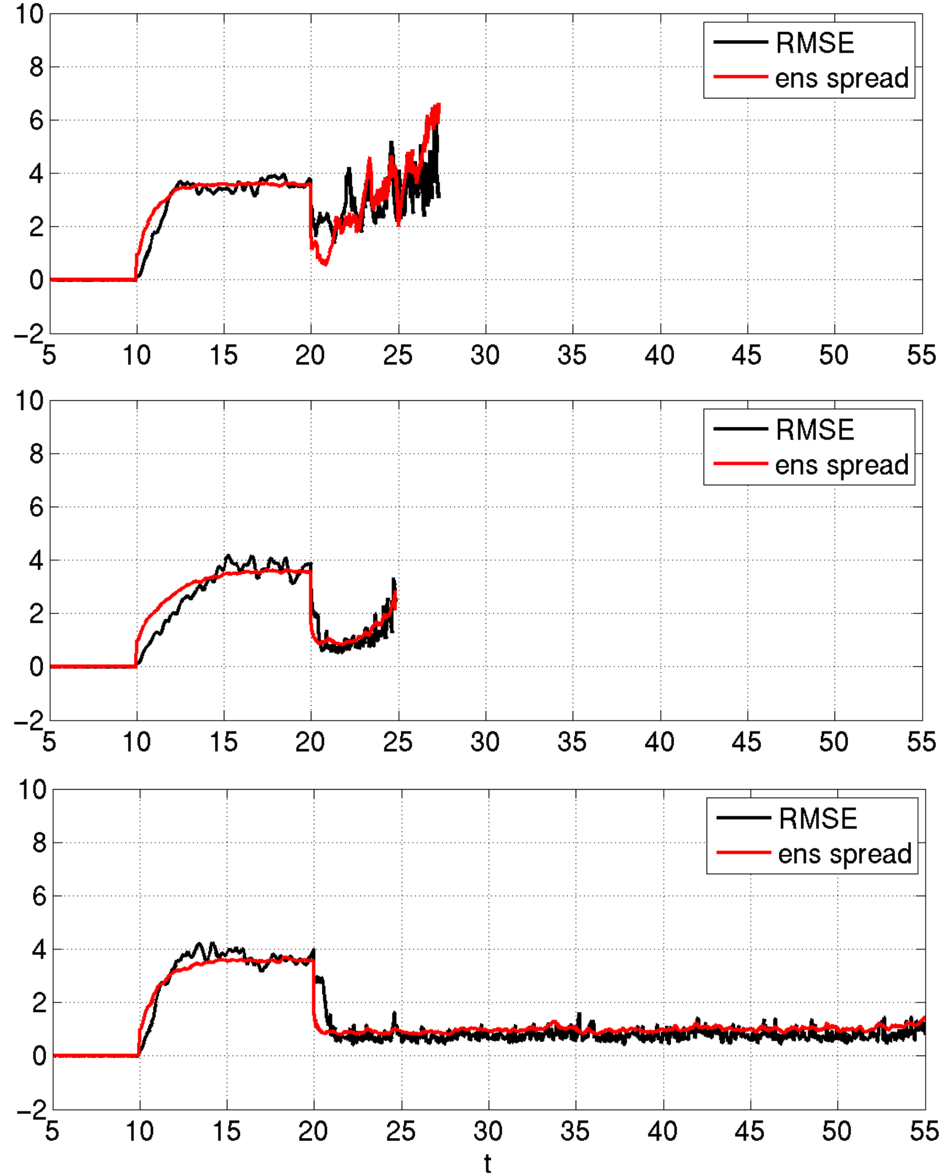


Randomly located observations

Adaptive RTPP

Adaptive inflation

Adaptive inflation (spatially varying)



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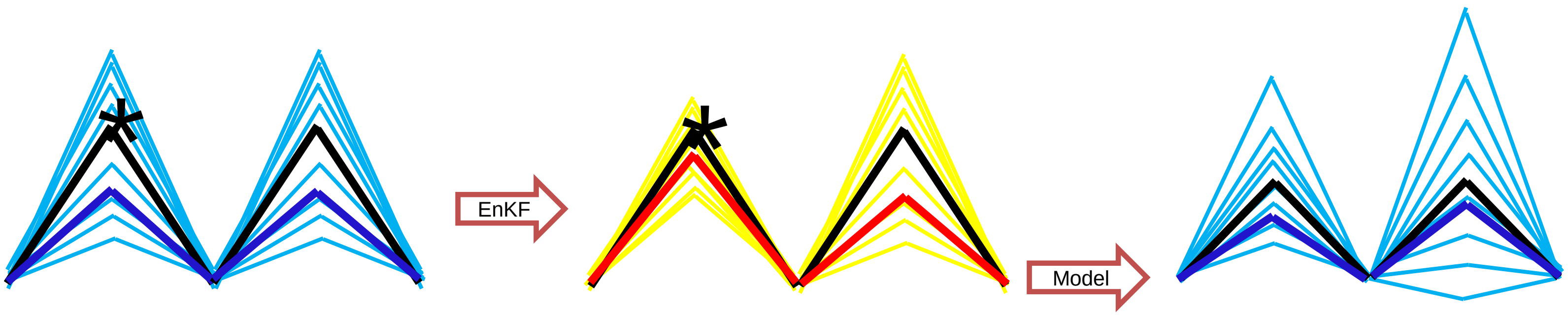

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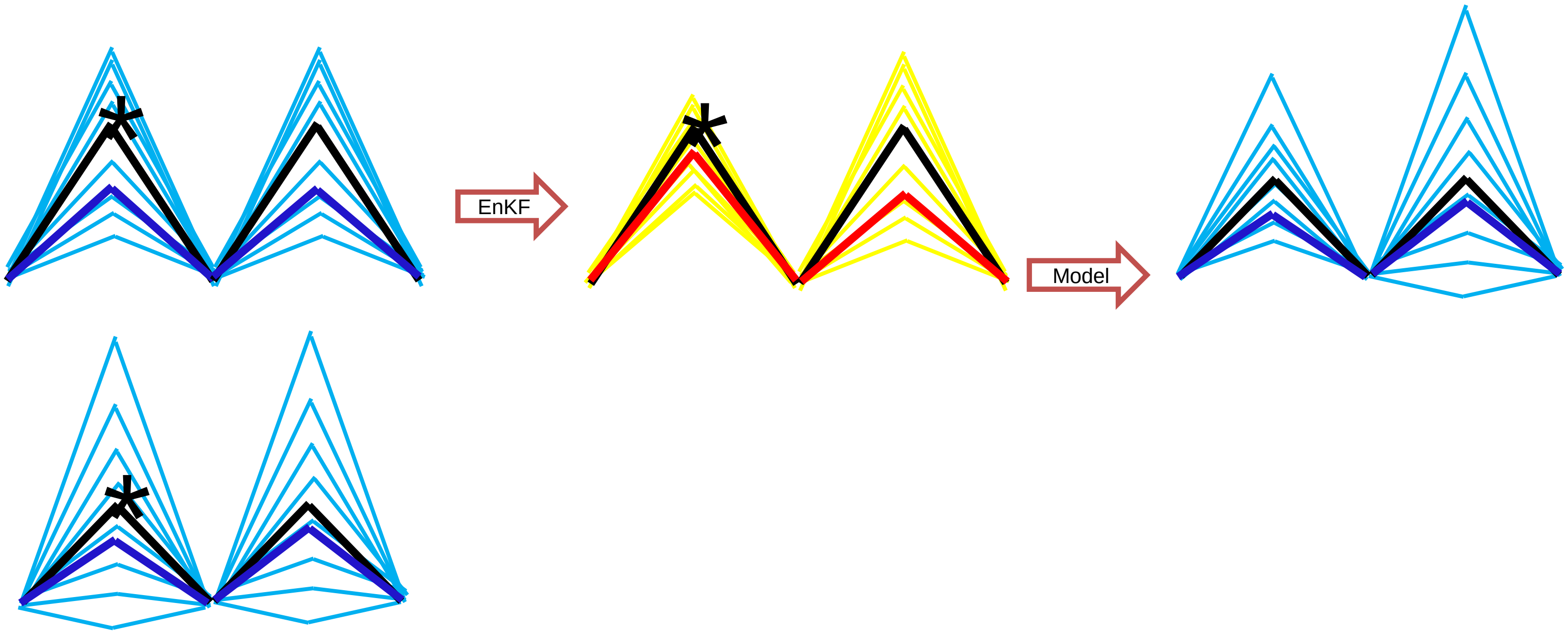
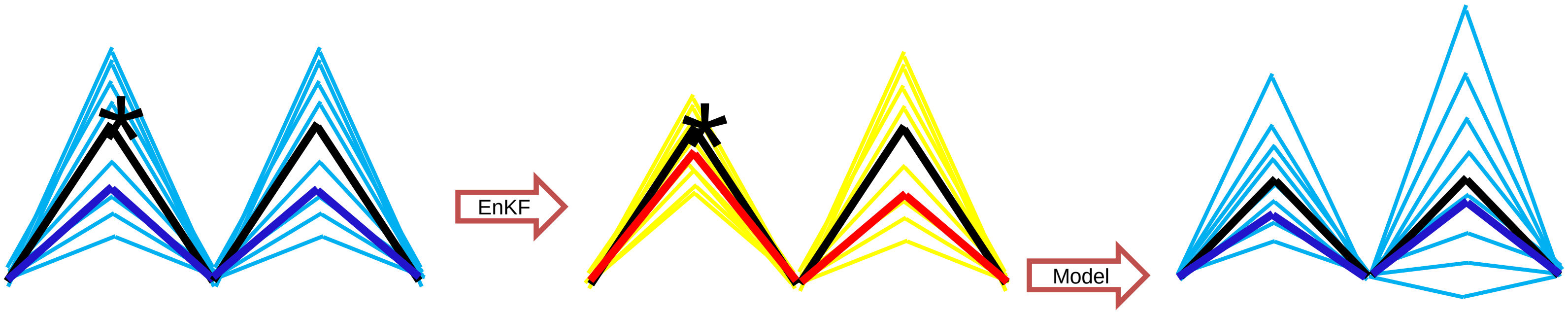
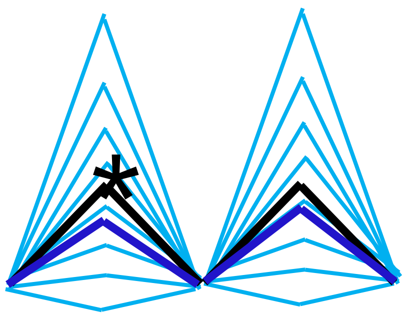


domain-averaged inflation

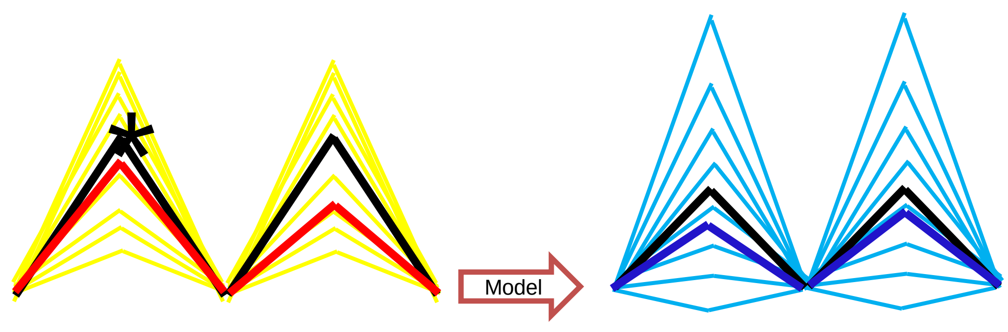
Randomly located observations





domain-averaged inflation

domain-averaged RTPP



Conclusion

verified some well-known data assimilation problems

adaptive methods can find the optimum λ, α values during EnKF cycle

randomly located observations cause trouble: need spatially varying methods

Further work: implementation of adaptive RTPP in atmospheric models