A simple state and observation network dependent multiplicative inflation algorithm

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Inflation algorithms (applied to posterior)

① Simple covariance inflation (a > 1)

$$\mathbf{x}_{\mathrm{a}}^{'} = \alpha \mathbf{x}_{\mathrm{a}}^{'}$$

② Relaxation to prior (0 < a < 1)

$$\mathbf{x}_{\mathrm{a}}^{'} = \alpha \mathbf{x}_{\mathrm{b}}^{'} + (1 - \alpha) \mathbf{x}_{\mathrm{a}}^{'}$$

③ State-dependent covariance inflation (a > 0)

$$\mathbf{x}_{\mathrm{a}}^{'} = \mathbf{x}_{\mathrm{a}}^{'} \sqrt{\alpha \frac{\sigma_{\mathrm{b}}^{2} - \sigma_{\mathrm{a}}^{2}}{\sigma_{\mathrm{b}}^{2}} + 1}$$

✓ ***Simple cov inflation*** as in Anderson and Anderson (1999)

$$\mathbf{x}_{\mathrm{a}}^{'} = \alpha \mathbf{x}_{\mathrm{a}}^{'}$$

- Pros:

• simple

- Cons:

• potential problems when observing network and/or dynamics are not homogeneous.

• doesn't add any new directions to ensemble.

✓ ***Relaxation to prior*** as in Zhang et al (2004)

$$\mathbf{x}_{\mathrm{a}}^{'} = \alpha \mathbf{x}_{\mathrm{b}}^{'} + (1-\alpha)\mathbf{x}_{\mathrm{a}}^{'}$$

- Pros:

- no inflation where there are no increments.
- more inflation where there are dense/accurate obs.
- retains growing structures in background.

- Cons:

- may converge to leading Lyapunov vector (introducing co-linearity to ensemble perturbations).
- removes part of the rotation in ensemble space introduced by analysis step.

✓ **State-dependent covariance inflation** (proposed)

$$\mathbf{x}_{\mathrm{a}}^{'} = \mathbf{x}_{\mathrm{a}}^{'} \sqrt{\alpha \frac{\sigma_{\mathrm{b}}^2 - \sigma_{\mathrm{a}}^2}{\sigma_{\mathrm{b}}^2} + 1}$$

- Motivation:

• sampling error largest where s_b/s_a is large (Sacher and Bartello 2008 MWR).

• model error is a larger fraction of background error in regions of dense/accurate obs (where s_b/s_a is large, Daley and Menard 1993 MWR).

• adaptively estimated inflation (Anderson 2009) looks like s_b/s_a

- Pros:

• no inflation where there are no increments.

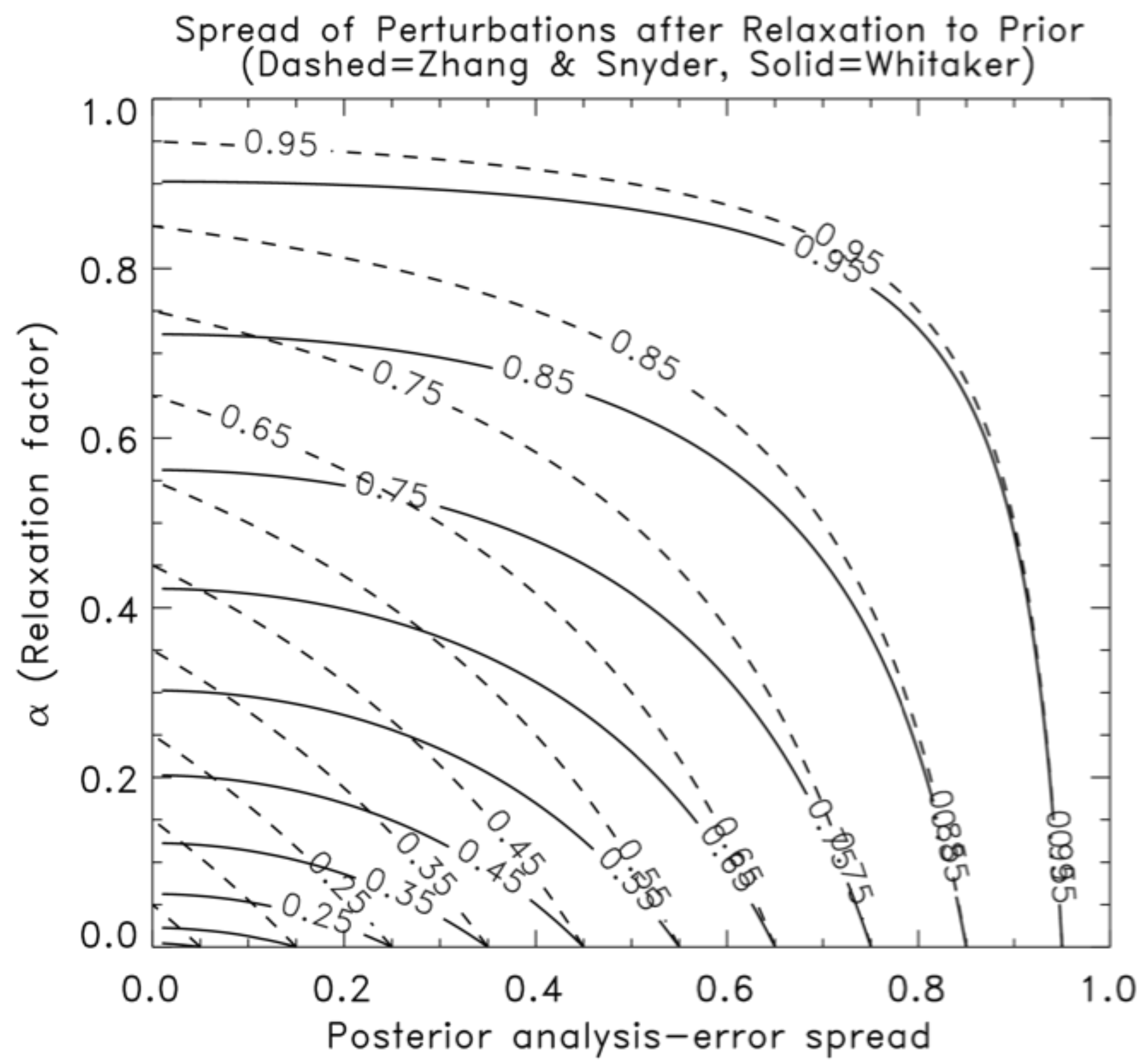
• more inflation where there are dense/accurate obs.

- Cons:

• potentially large spatial gradients in inflation may disrupt growing structures.

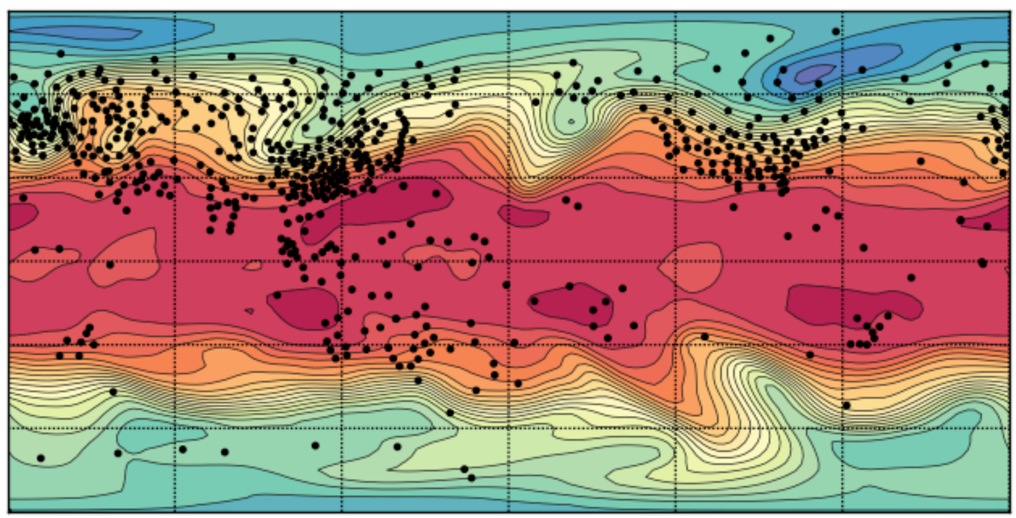
Effect of inflation on posterior spread when prior spread = 1

- For small a and small posterior spread, new method increases spread more.
- Posterior spread will be more uniform in new method when prior spread similar but spread after assimilation has strong spatial variations (solid contours have flatter slope in upper left quadrant).



Tests with a simple GCM

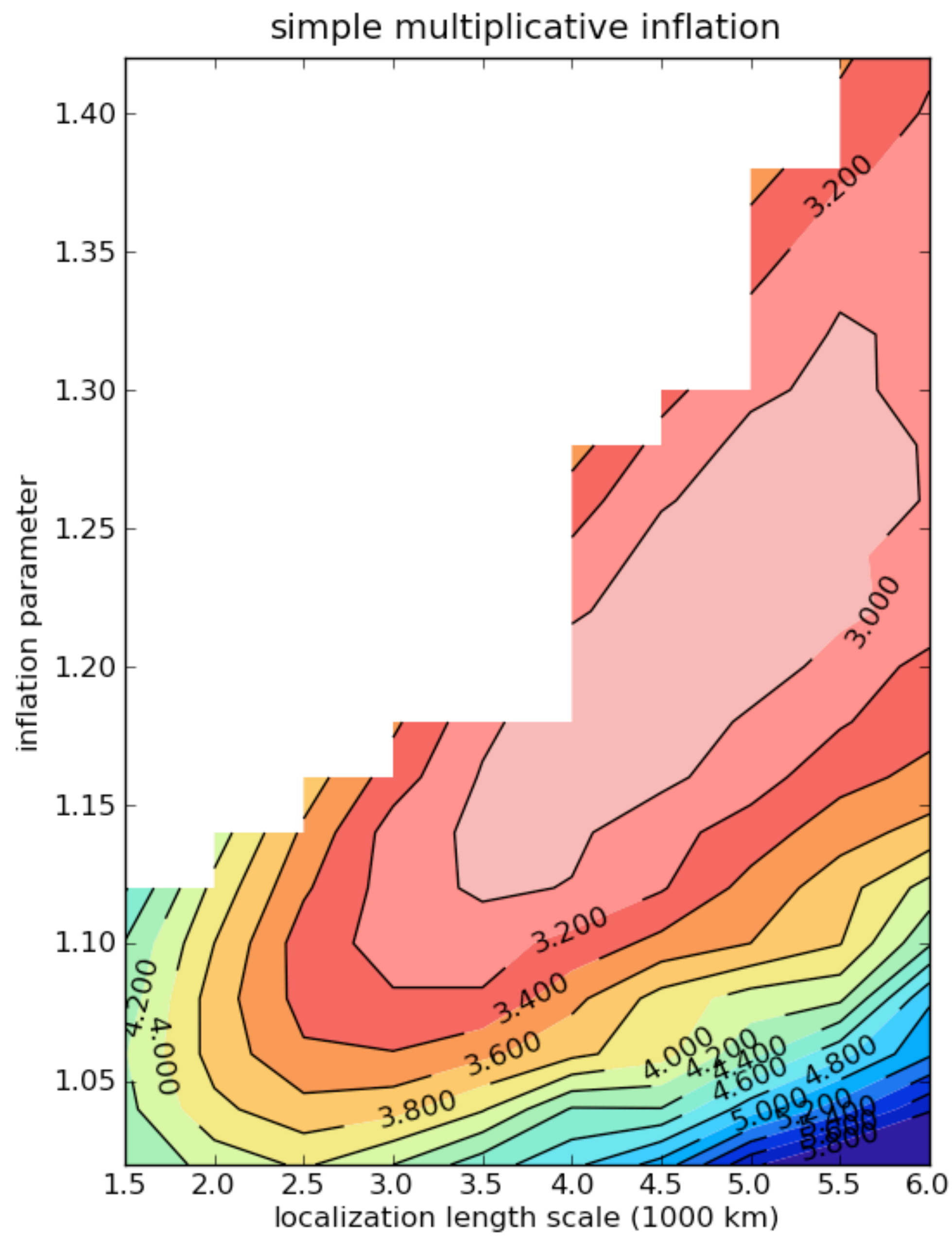
- 2-level PE model on a sphere (Lee and Held, 1993 with parameters as in Hamill and Whitaker, 2010).
- 603 obs of T, verticaly averaged wind with 1^oK, 1 ms^{-1} error at sonde locations sampled from a T42 nature run. Assimilation with 20 member EnKF at T31 resolution every 12-h, with Gaspari-Cohn type localization.
- Error measured in vertically averaged zonal wind over 150 days of assimilation.



Simple covariance inflation

min RMS error of ~2.9 ms^{-1} at r=1.18, L=4000km

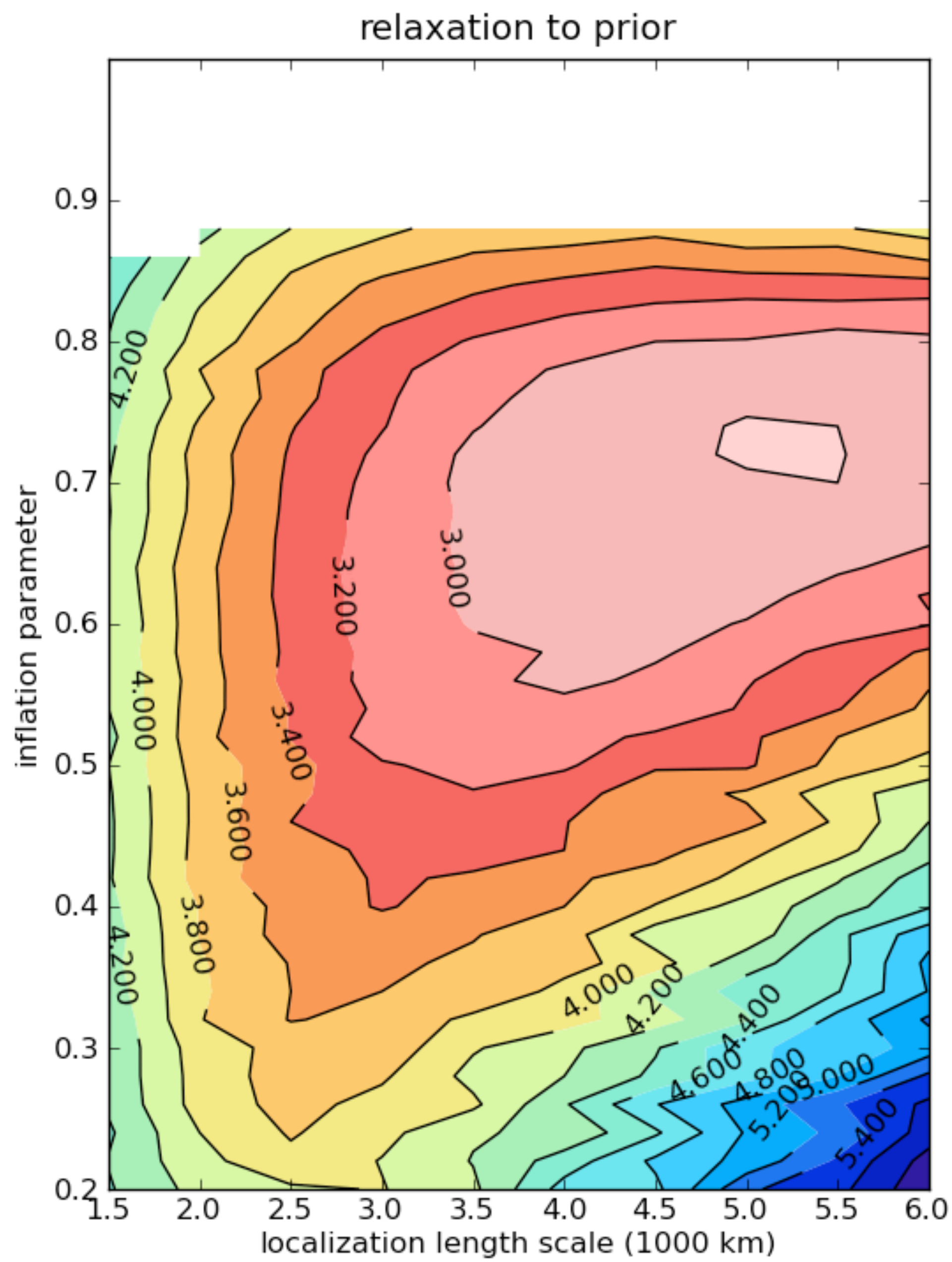
- Goes bad quickly if inflation is set to value larger than optimal.



Relaxation to prior

min RMS error of ~2.8 ms^{-1} at
a=0.74, L=5000km

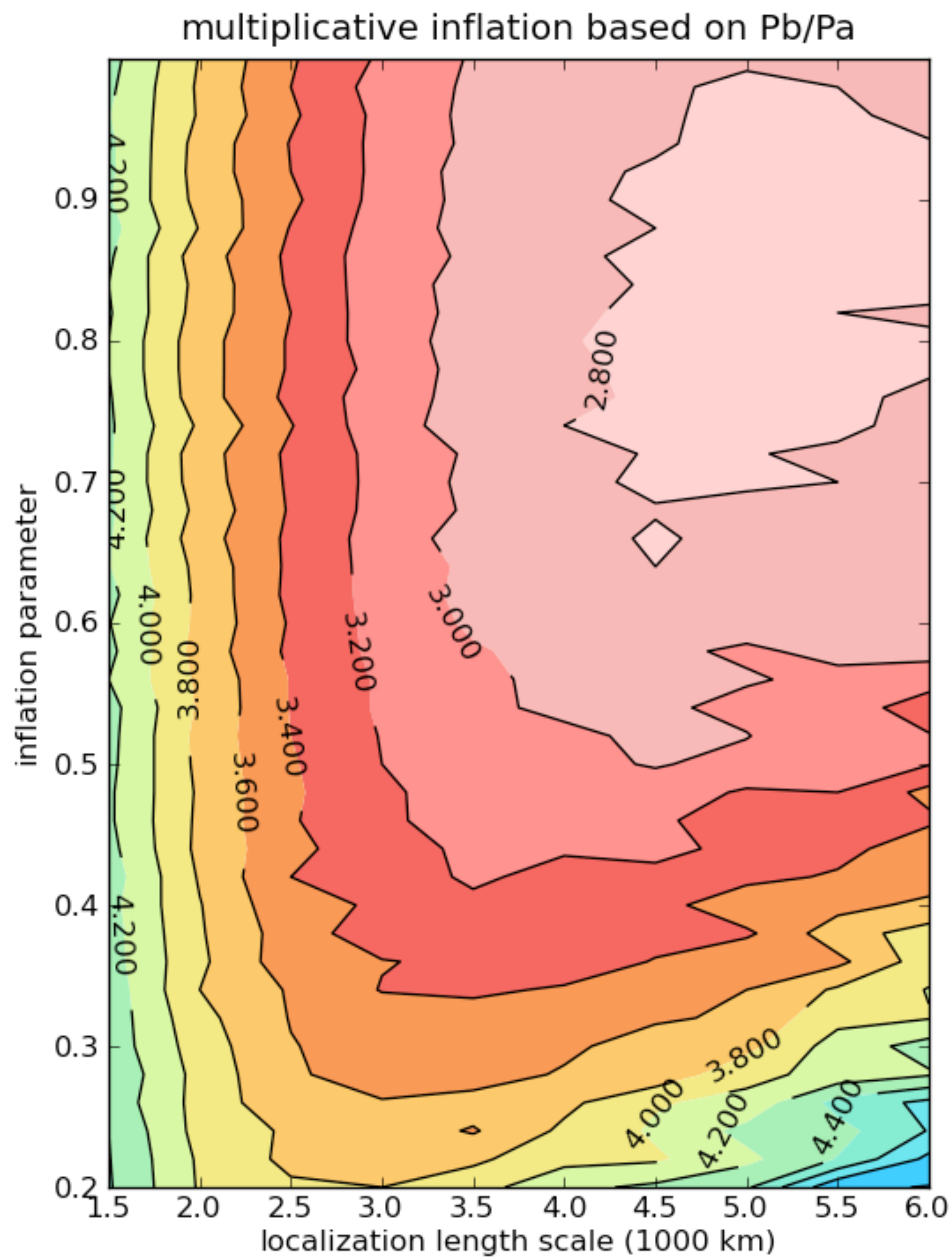
- Slightly lower error than simple inflation.
- Stable over most of of parameter space, but goes bad quickly when a exceeds optimal value.



State Dependent Multiplication inflation

min RMS error of ~2.7 ms^{-1} at a=0.92, L=5500km

- Lowest error (by a hair).
- Stable over the whole parameter space.
- Large localization length scale at minimum.



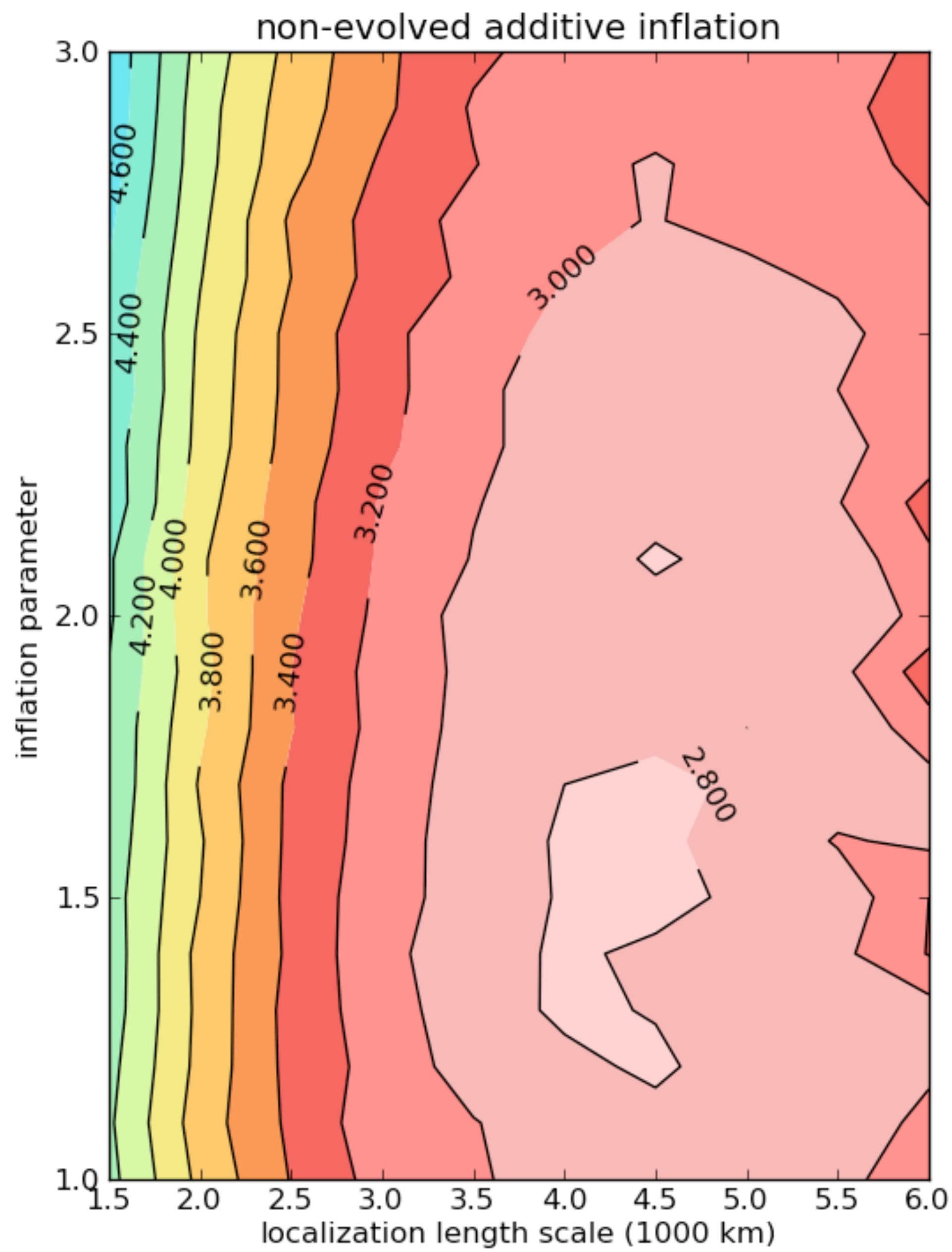
Additive Inflation

- Random samples of model error estimates added to each ensemble member after analysis.
- Here we use errors of T31 12-h forecasts initialized from truncated T42 truth run (randomly chosen, not flow dependent, scaled by constant b). This is a best-case scenario – can't do this in the real world.
  - "non-evolved" (added to posterior ensemble)
  - "evolved" (added to ensemble mean analysis from 12-h prior, then integrated 12-h and added to posterior ensemble). This conditions perturbations to dynamics, adds some flow dependence (Hamill and Whitaker, 2010).
- Pros:
  - Adds some new directions to ensemble.
- Cons:
  - Must pre-generate a sample.
  - Don't know what to draw the sample from.

Additive Inflation

min RMS error of ~2.7 ms^{-1} at b=1.5, L=4500km

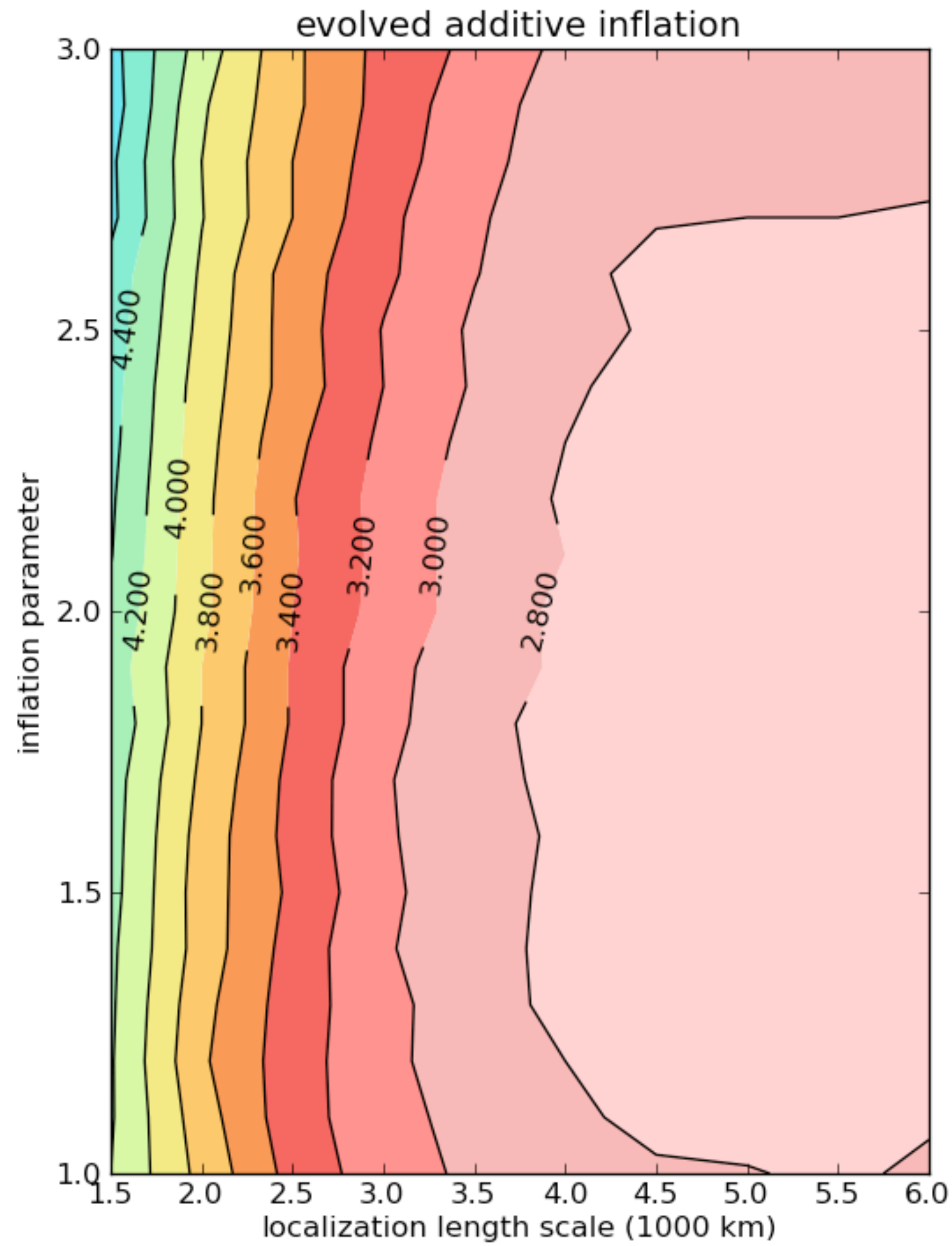
- Performance comparable to multiplicative inflation.



Evolved Additive Inflation

min RMS error of ~2.7 ms^{-1} at b=2.0, L=5000km

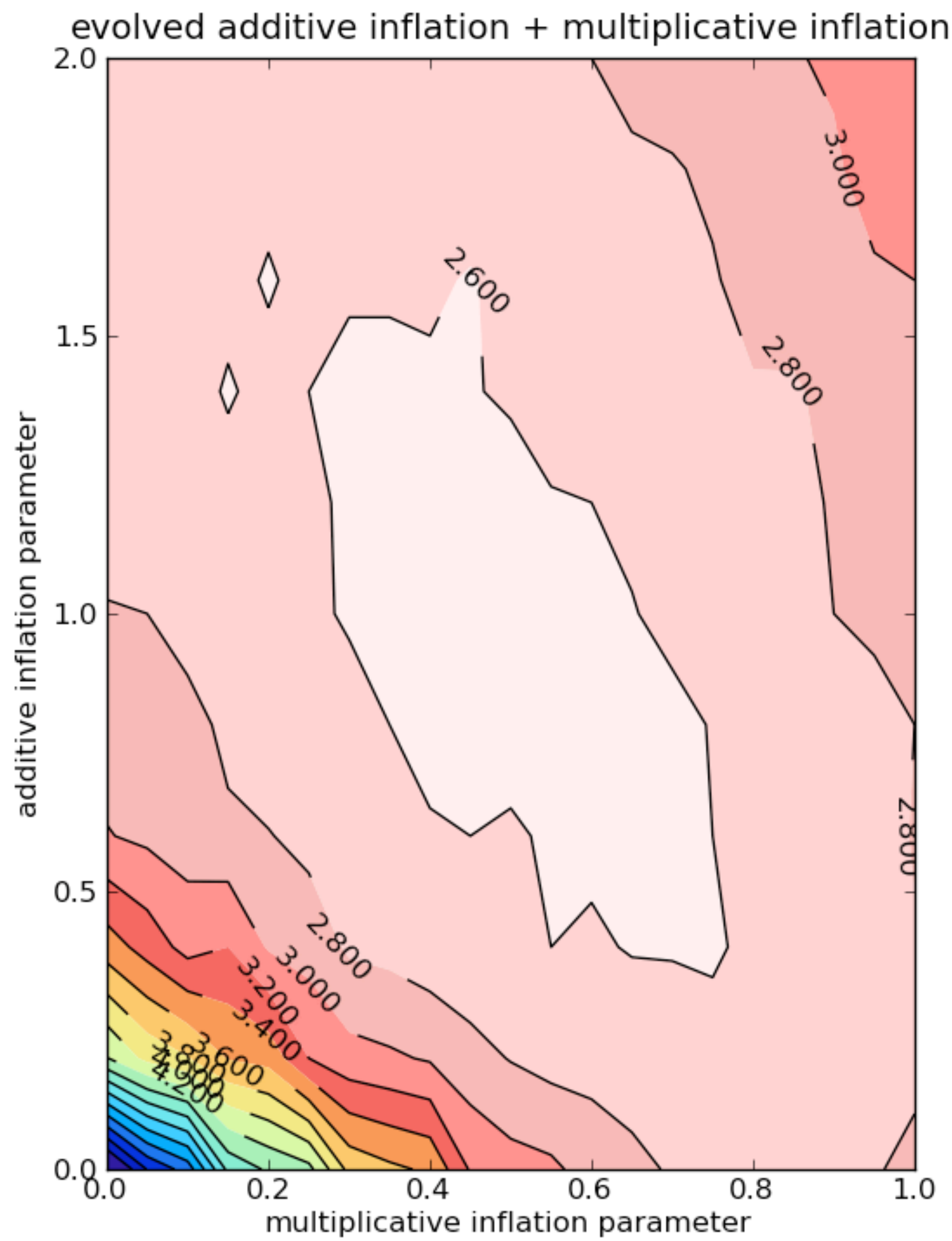
- Conditioning the additive noise to the dynamics helps.



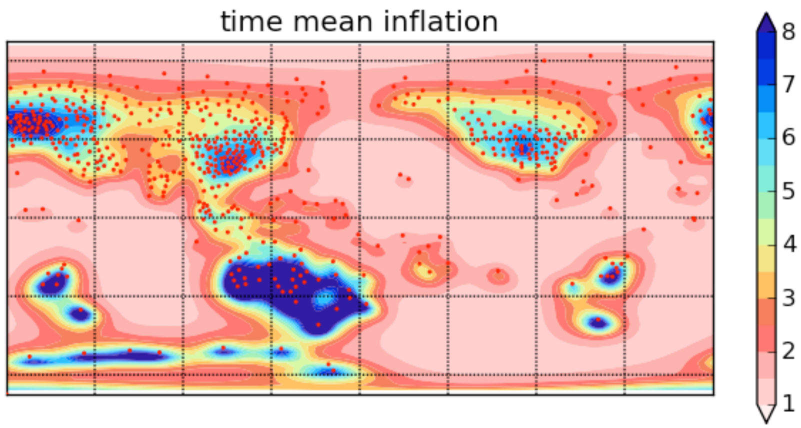
Evolved Additive Inflation plus Multiplication inflation fixed localization at 5000 km

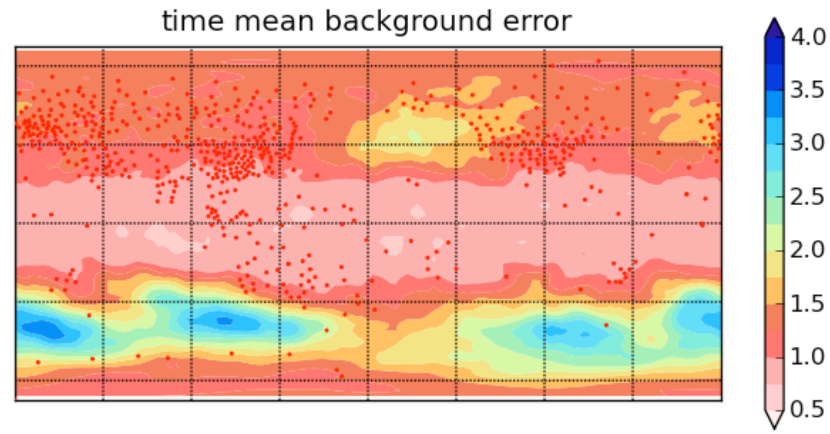
min RMS error of ~2.5 ms^{-1} at a=0.5, b=1.0

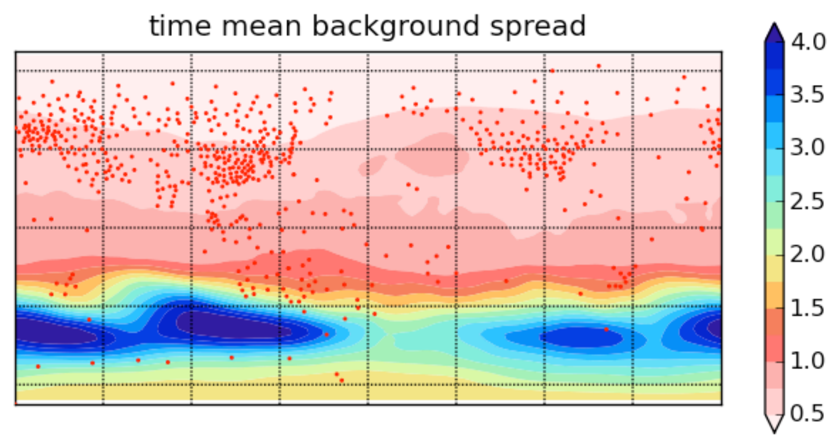
- Better than multiplicative or additive inflation alone.
- Suggests additive and multiplicative inflation are accounting for different error sources (model and assimilation).
- Similar result obtained using random 12-h model tendencies for additive inflation.



Time mean statistics state-dependent multiplicative inflation (a=0.92, L=5500km)

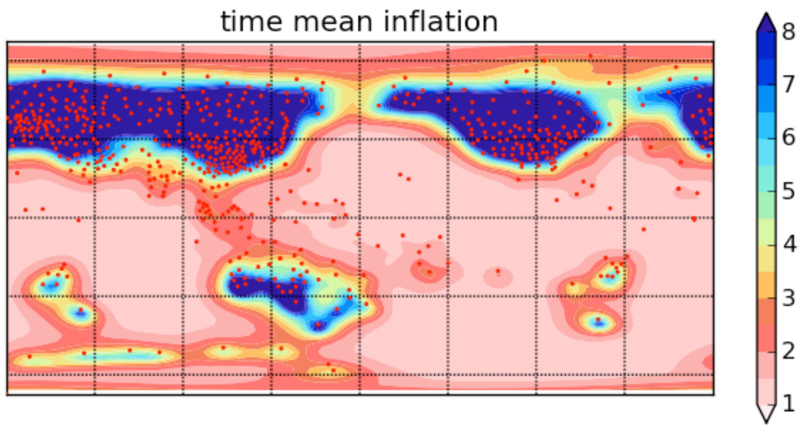


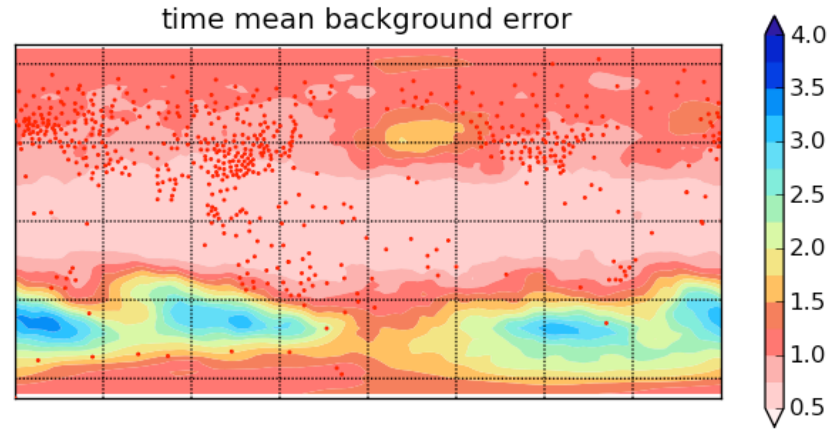


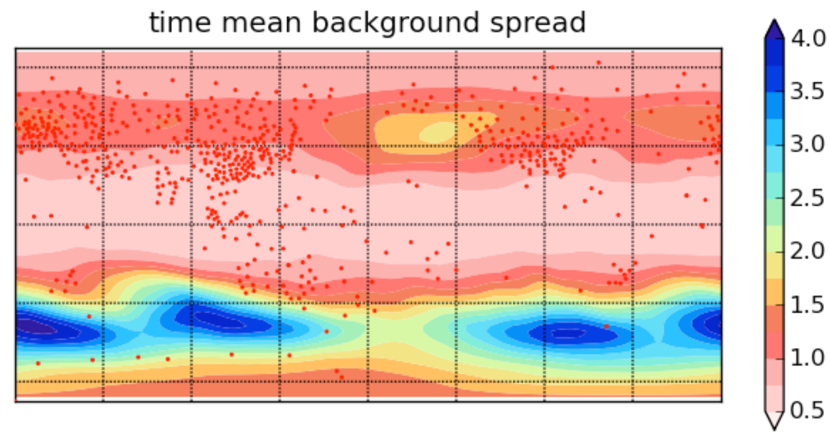


- ***Larger inflation where obs are dense, background uncertainity large.***
- ***Spread too small (large) in NH (SH).***

Time mean statistics for combined multiplicative and evolved additive inflation (a=0.5, b=1.0, L=5000km)







- ***Error reduced in observation dense NH.***
- ***Multiplicative inflation is actually larger (even though parameter a is smaller – due to increased background spread).***

Conclusions

- New state-dependent multiplicative inflation is simple to implement and works well, especially when dynamics and/or observing network are spatially inhomogenous.
- A combination of ‘evolved’ additive inflation and multiplicative inflation works best.
  - Multiplicative inflation handles observing network dependent assimilation errors.
  - Additive inflation handles model errors that are independent of observing network (may be preferable to treat this in the model stochastically?).

References

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