Initialization and Direct Minimization of the Cost Function in the LETKF

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Motivation

Issues on which EnKF still have to catch up with 4D-Var:

- Initialization of the analysis increment, internally in the analysis, e.g., by a weak (balance) constraint
- Direct minimization of the cost function, which allows for
  - The introduction of balance constraints
  - Accounting for nonlinearities in the observation operator
  - Variational QC
  - Non-Gaussian Error Statistics

Current Approach for Initialization (Finalization) in EnKF

Summary of Current Approach

- An (external) digital filter (finalization) is applied to all background ensemble members
- It is hoped that the analysis will be balanced, because the analysis increment is obtained in the space of ensemble perturbations that are spanned by the filtered ensemble members
- We all do it, because it improves analysis accuracy

Potential Problems with Current Approach:

- It filters the full model solution, not only the increment: it affects the analysis not only through the analysis increment, but also through the background mean
- The filtered background is not constrained by the observations at the previous time

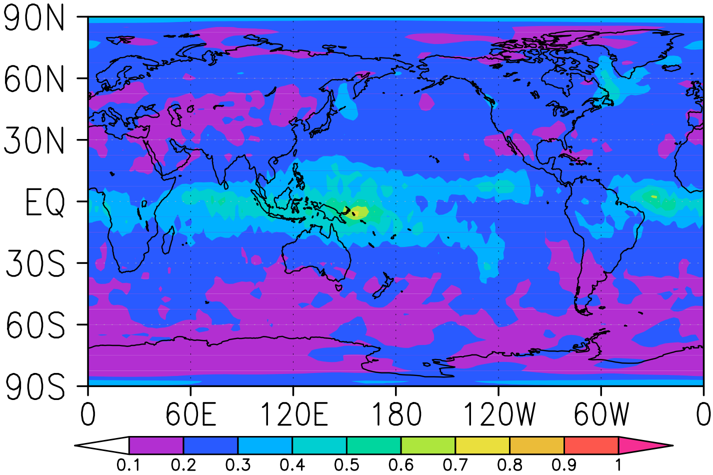
Illustration of Problem with The Semidiurnal Tidal Wave

Semidiurnal Tidal Wave:

- A 1 *hPa* amplitude semidiurnal oscillation in the surface pressure in the Tropics
- The most regular periodic motion in the atmosphere
- A response of the atmosphere to the excitation due to the absorption of solar radiation by ozone in the stratosphere. This response travels in the form of a gravity wave (Chapman and Lindzen 1970).
- Any model that has stratospheric ozone as prognostic variable can easily handle it

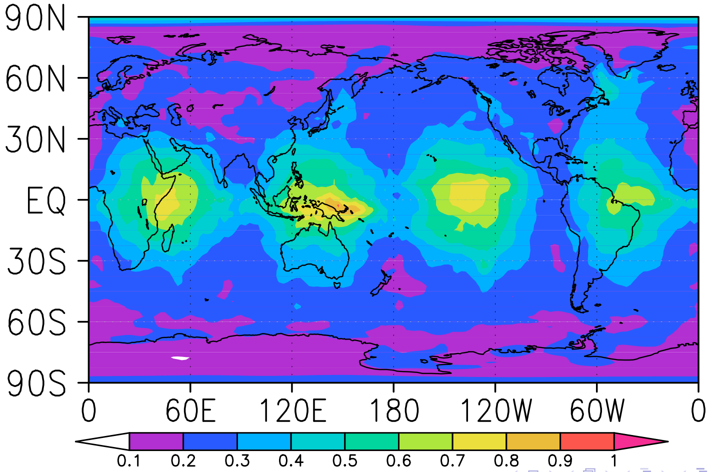
Results are from **Satterfield and Szunyogh (2010, MWR)**–simulated observations of all atmospheric variables in randomly selected atmospheric columns (10% observation coverage)

Time mean absolute error of surface pressure analysis with no digital filter



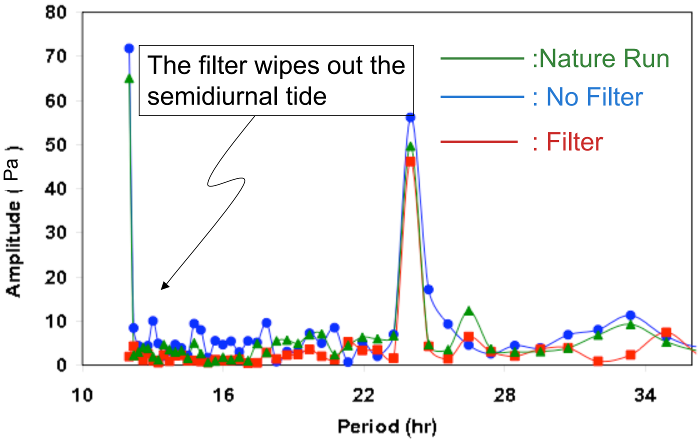
Time mean absolute error of surface pressure analysis with digital filter

The error pattern suggests that we lost the semidiurnal tidal wave from the analysis



Fourier Spectrum of the Surface Pressure Analysis

The Fourier analysis confirms our suspicion



Digital Filter as a Weak Constraint

Cost function with balance constraint:

$$J(\delta \mathbf{x}) = J_b(\delta \mathbf{x}) + J_o(\delta \mathbf{x}) + J_c(\delta \mathbf{x})$$

where the **analysis increment** is $\delta \mathbf{x} = \bar{\mathbf{x}}^a - \bar{\mathbf{x}}^b$
Introduction of the digital filter as a weak constraint (**Gustafsson 1992, Polavarapu et al. 2000, Gauthier and Thepaut 2001**):

$$J_c(\delta \mathbf{x}) = (\delta \mathbf{x} - \hat{\delta} \mathbf{x})^T \mathbf{Q} (\delta \mathbf{x} - \hat{\delta} \mathbf{x}),$$

Q is a metric and filtered increment, $\hat{\delta}\mathbf{x}$, is defined by

$$\hat{\delta}\mathbf{x} = \sum_{i=0}^{I} \alpha_i \delta \mathbf{x}(t_i),$$

Remark: In most EnKF implementation $J_o(\delta \mathbf{x})$ is computed in a 4-D fashion

How to Introduce the Weak Constraint into the LETKF?

In the LETKF $\delta\mathbf{x} = \mathbf{X}^b\mathbf{w}$, where $\mathbf{X}^b$ is the matrix of ensemble background perturbations and $\mathbf{w}$ is a vector of weights. We can write the constraint as

$$J_c(\mathbf{w}) = \mathbf{w}^T(\mathbf{X}^b - \hat{\mathbf{X}}^b)^T\mathbf{Q}(\mathbf{X}^b - \hat{\mathbf{X}}^b)\mathbf{w},$$

We add this term to one of the following two cost functions proposed in **Hunt et al. (2007)**

Option 1: (Nonlinear observation operator)

$$J_{b+o}(\mathbf{w}) = (k-1)\mathbf{w}^T\mathbf{w} + [\mathbf{y}^o - H(\bar{\mathbf{x}}^b + \mathbf{X}^b\mathbf{w})]^T\mathbf{R}^{-1}[\mathbf{y}^o - H(\bar{\mathbf{x}}^b + \mathbf{X}^b\mathbf{w})].$$

Option 2: (**Linearized observation operator:** quadratic cost function)

$$\begin{aligned} J_{b+o}(\mathbf{w}) &= (k-1)\mathbf{w}^T\mathbf{w} \\ &+ [\mathbf{y}^o - \bar{\mathbf{y}}^b - \mathbf{Y}^b\mathbf{w}]^T\mathbf{R}^{-1}[\mathbf{y}^o - \bar{\mathbf{y}}^b - \mathbf{Y}^b\mathbf{w}] \end{aligned}$$

Option I

Computation Algorithm:

1. Compute $\bar{\mathbf{w}}^a$, the weight that defines the mean analysis, by the direct minimization of the cost function
2. Compute the analysis covariance matrix $\mathbf{P}^a$ as the inverse of the Hessian of the cost function at $\bar{\mathbf{w}}^a$.
3. Compute the weights for the analysis perturbation from $\mathbf{P}^a$ the usual way

Pros:

- It takes into account the nonlinearity in the observation operator
- Nonlinear minimization can be done in parallel for each grid point: most likely cheaper than a 4D-Var using global direct minimization

Cons:

- Complicates coding and code maintenance: H cannot be pre-computed outside the analysis

Option I

Computation Algorithm: Same as LETKF except that the analysis error covariance matrix has to be modified as

$$\mathbf{P}^a = [(k-1)\mathbf{I} + (\mathbf{Y}^b)^T\mathbf{R}^{-1}\mathbf{Y}^b + (\mathbf{X}^b - \hat{\mathbf{X}}^b)^T\mathbf{Q}(\mathbf{X}^b - \hat{\mathbf{X}}^b)]^{-1}.$$

Pros:

- Can be implemented as a simple change in the LETKF code

Remarks:

- It does not require a direct minimization, but it may be useful to code it with direct minimization if we plan to add non-quadratic terms in the future, e.g., to introduce a non-Gaussian backgroun term as in **Harlim and Hunt (2007)**

Conclusions

- Initialization is an area where EnKF schemes can be (easily?) improved
- We have an idea of how to do it, but no results to show that it actually works
- Direct minimization holds promise of further improvements for EnKF. But, code implementation may be challenging

A Real-Life Tale for Young Scientists: Sometimes it pays off to do research in science and technology...

Rudolf Kalman Receives the National Medal of Science and Technology on October 7, 2009: