An adaptive covariance inflation method with the LETKF

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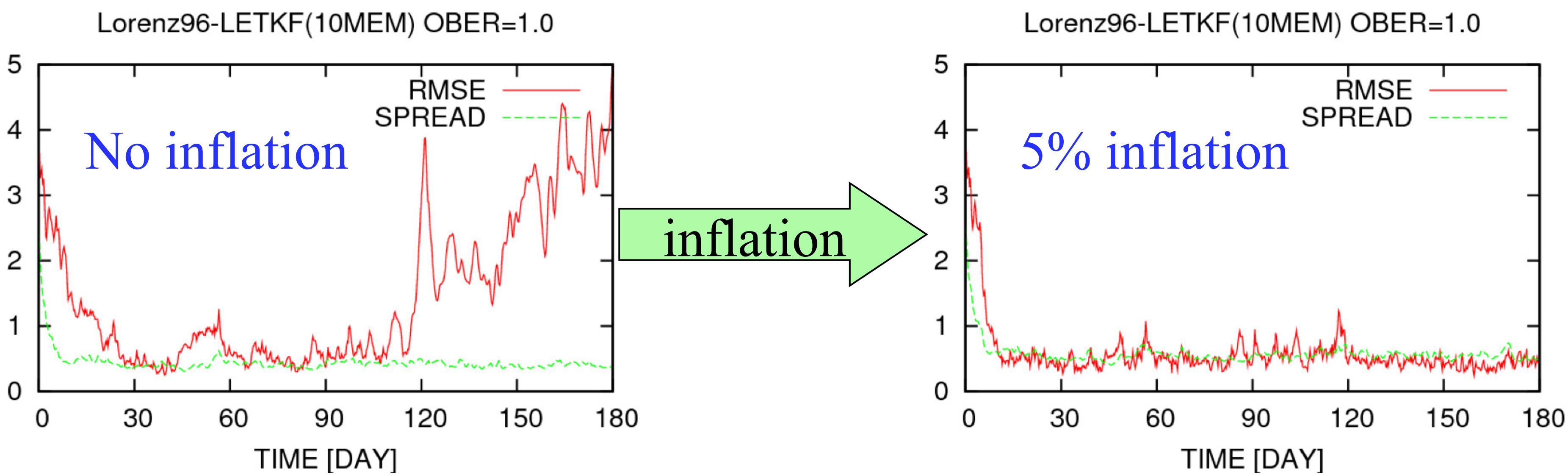
Covariance inflation (e.g., Houtekamer and Mitchell 1998)

Empirical treatment for…
- treating covariance underestimation

Error covariance is underestimated due to *various sources of imperfections*:

- limited ensemble size
- nonlinearity
- model errors

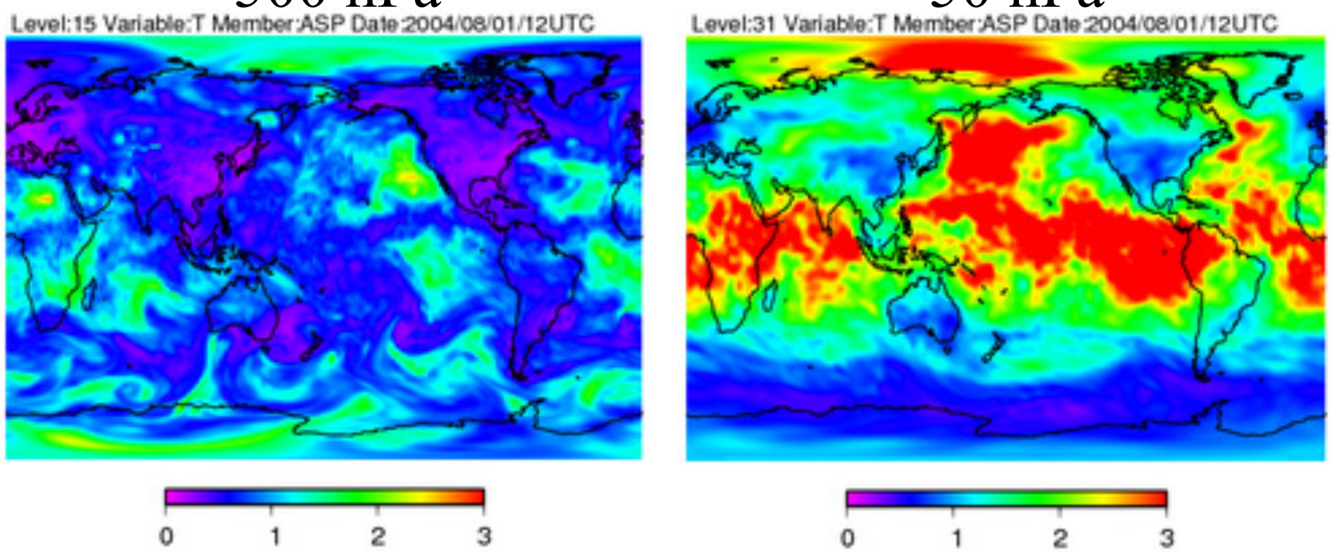


Difficulties of inflation

Fixed covariance inflation has difficulties such as…
- ensemble spread exaggerates observing density pattern
- should depend on (x, y, z, t); tuning is very difficult

Temperature spread of JMA's global LETKF
w/ fixed multiplicative inflation

~500 hPa ~50 hPa



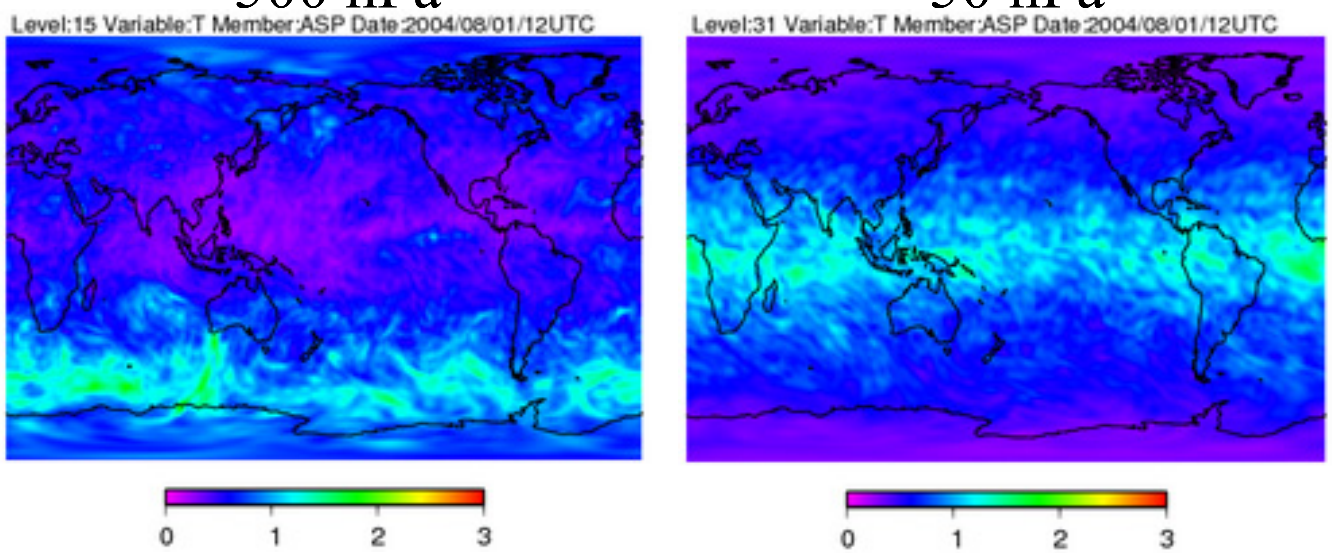
adapted from Miyoshi et al. (2010)

Additive inflation

- introduces new directions to span the error space
- solves the problem of the spread pattern
- has difficulties in obtaining reasonable additive noise

Temperature spread of JMA's global LETKF
w/ additive inflation

~500 hPa ~50 hPa



adapted from Miyoshi et al. (2010)

Adaptive inflation

Anderson (2007; 2009) developed an adaptive inflation algorithm using a hierarchical Bayesian approach.

We follow ***Li et al. (2009)*** and use the statistical relationship derived by ***Desroziers et al. (2005)***:

$$d_b^a = Hx^a - Hx^b = H\,dx^a = HKd_b^o$$

$$\left\langle d_b^a d_b^{o\,T} \right\rangle = HP^f H^T (HP^f H^T + R)^{-1} \underline{\left\langle d_b^o d_b^{o\,T} \right\rangle}$$

$$\boxed{\left\langle d_b^a d_b^{o\,T} \right\rangle = HP^f H^T}$$

using $\left\langle d_b^o d_b^{o\,T} \right\rangle = HP^f H^T + R$

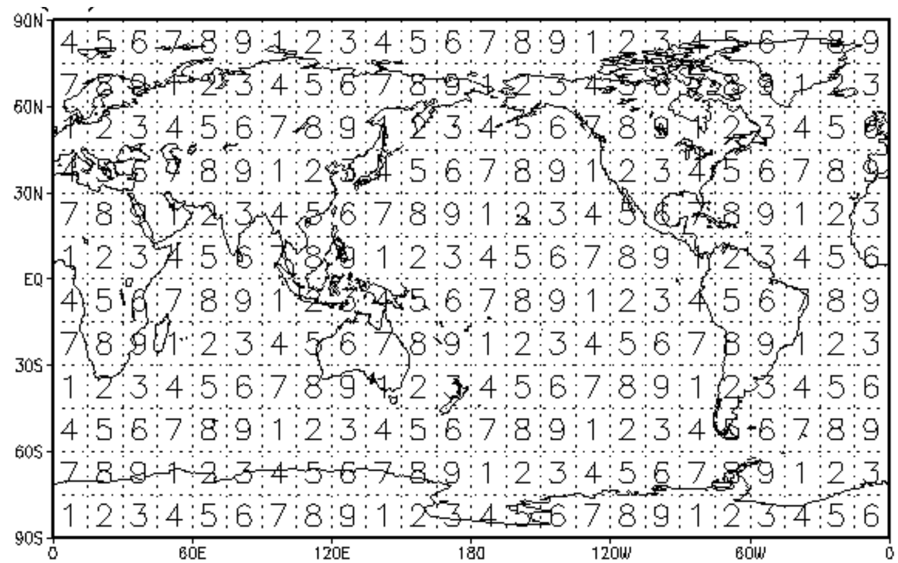
In the EnKF, $P^f \leftarrow \underline{\rho \circ P^f}$ This is important!!

i.e., $\left\langle d_b^a d_b^{o\,T} \right\rangle = H(\rho \circ P^f)H^T$

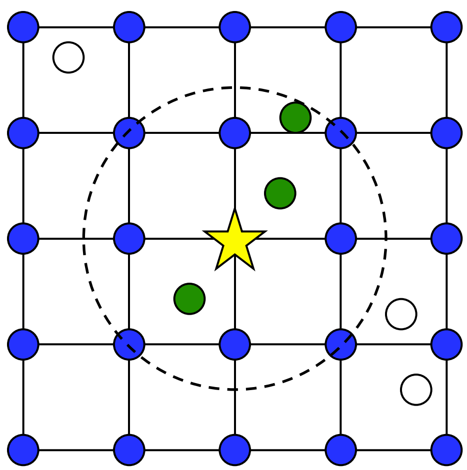
We estimate the inflation parameter: $\boxed{\alpha = \dfrac{tr\left\langle d_b^a d_b^{o\,T} \right\rangle}{tr\left(H(\rho \circ P^f)H^T\right)}}$

LETKF algorithm (Hunt et al. 2007)

Local Ensemble Transform Kalman Filter

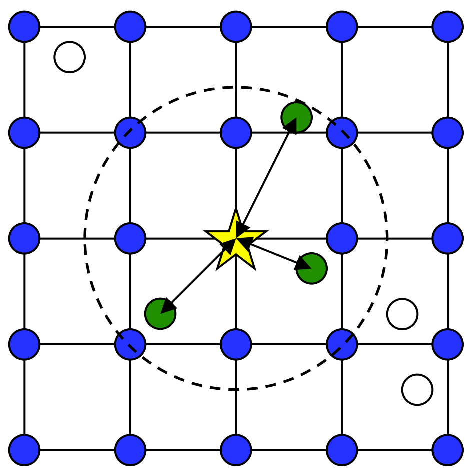


Each grid point is treated independently.



Multiple observations are treated simultaneously.

Adaptive inflation in the LETKF



Analysis of the *i*-th variable:

$$\underset{(N\times m)}{\mathbf{x}_i^a} = \overline{\mathbf{x}}_i^f \mathbf{1}_{1\times m} + \underset{(N\times m)}{\delta\mathbf{x}_i^f}\, \underset{(m\times m)}{\mathbf{T}_i}(\underline{\delta\mathbf{Y}_i^f, \mathbf{R}_i, \mathbf{d}_i})$$

$$\mathbf{R}_i \leftarrow \tilde{\rho}_i^{-1} \circ \mathbf{R}_i$$

R-localization, Hunt et al. (2007)

When computing $\mathbf{T}_i$, we compute the following statistics simultaneously:

$$\alpha_i = \frac{tr\left[\left\langle d_b^a d_b^{o\,T} \right\rangle \underline{(\tilde{\rho}_i^{-1} \circ \mathbf{R}_i)^{-1}}\right]}{tr\left[\tilde{\rho}_i \circ (\delta\mathbf{Y}_i^f \delta\mathbf{Y}_i^{f\,T}) \underline{(\tilde{\rho}_i^{-1} \circ \mathbf{R}_i)^{-1}}\right]}$$

Normalization factor

3-dimensional field of inflation

Time smoothing

Due to the limited sample size in the local region at a single time step, it is essential to apply time smoothing to include more samples in time.

We use the Kalman filter approach.

$$\boxed{\alpha_t = \frac{\sigma_o^2 \alpha_{t-1} + \sigma_b^2 \alpha_o}{\sigma_o^2 + \sigma_b^2}}$$

For example, $\sigma_b = 0.002$

a tuning parameter that determines the strength of time smoothing

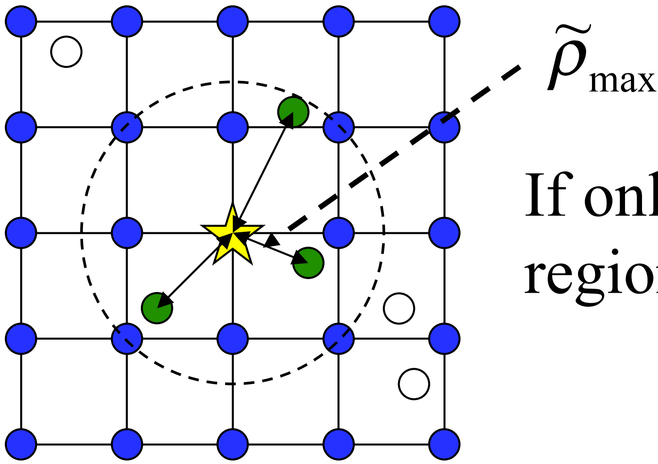
$$\sigma_o = \frac{1}{\sqrt{p}}$$

i.e., more samples, more reliable.

p denotes the number of observations (i.e., sample size)

For example, when $p = 100$, $\sigma_o = 0.1$

Further consideration



If only "far" observations exist in a local region, the statistics would be less reliable.

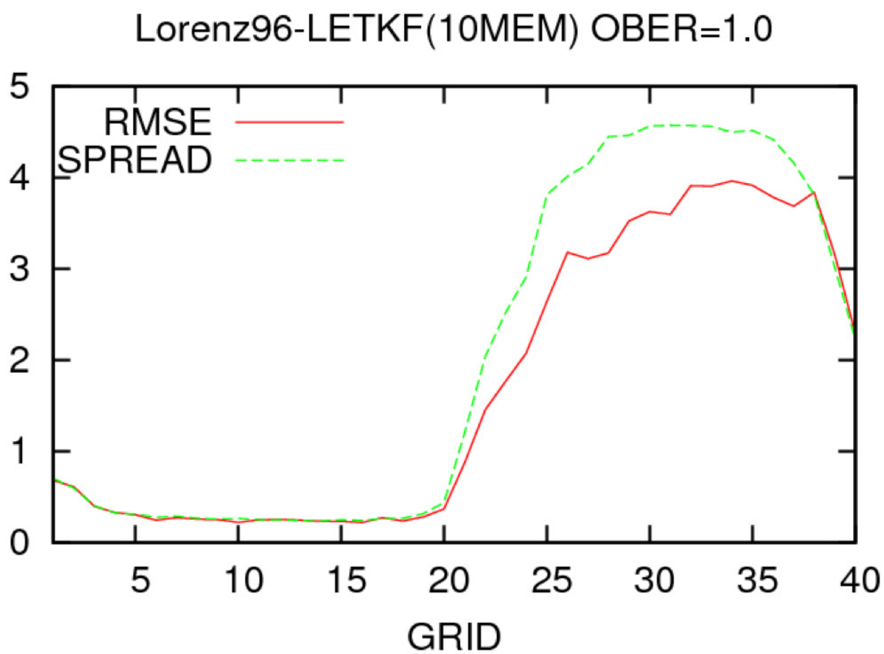
Considering the maximum localization weight of the local observations, we further modify the uncertainty of the estimated inflation value:

$$\sigma_o = \frac{1}{\tilde{\rho}_{\max}\sqrt{p}}$$

Localization weighting function for the closest observation

Results with the Lorenz 96 model

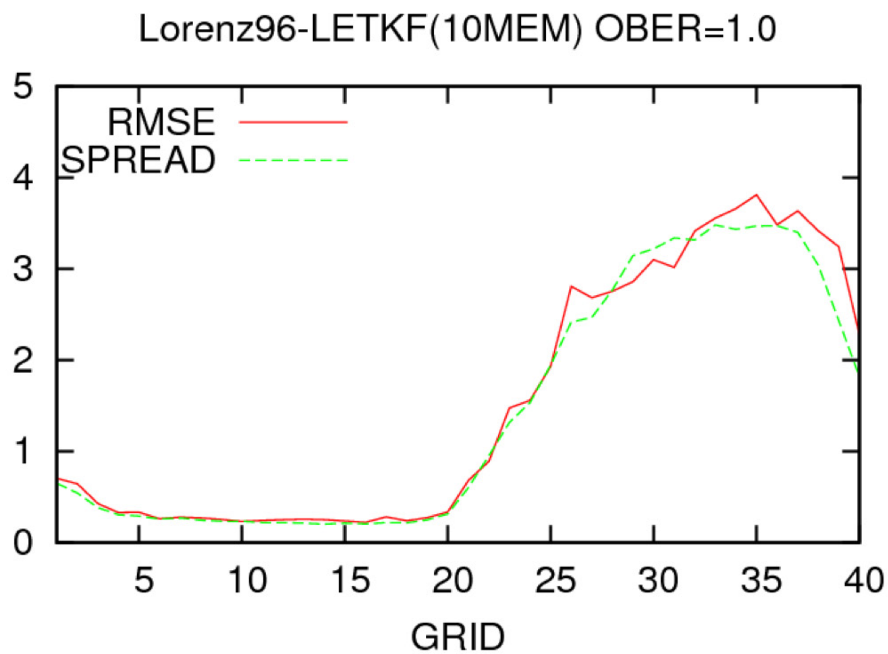
5% inflation



Observed

No obs

adaptive inflation



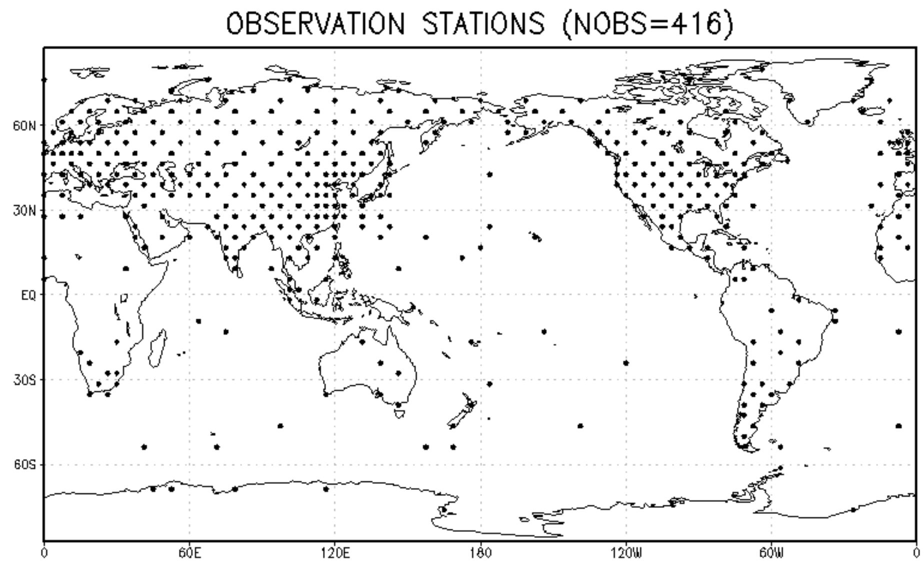
Observed

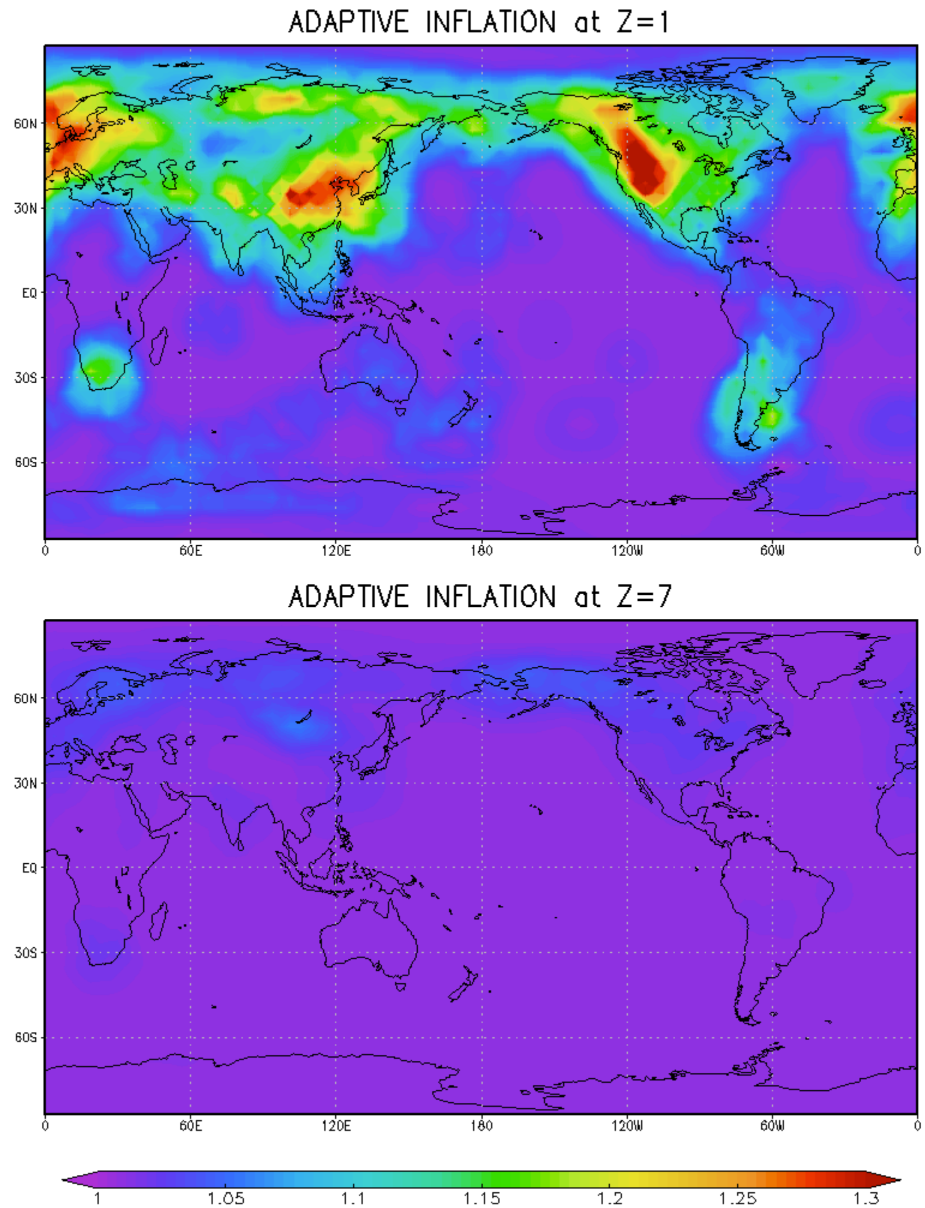
No obs

6-month average after 6-month spin-up

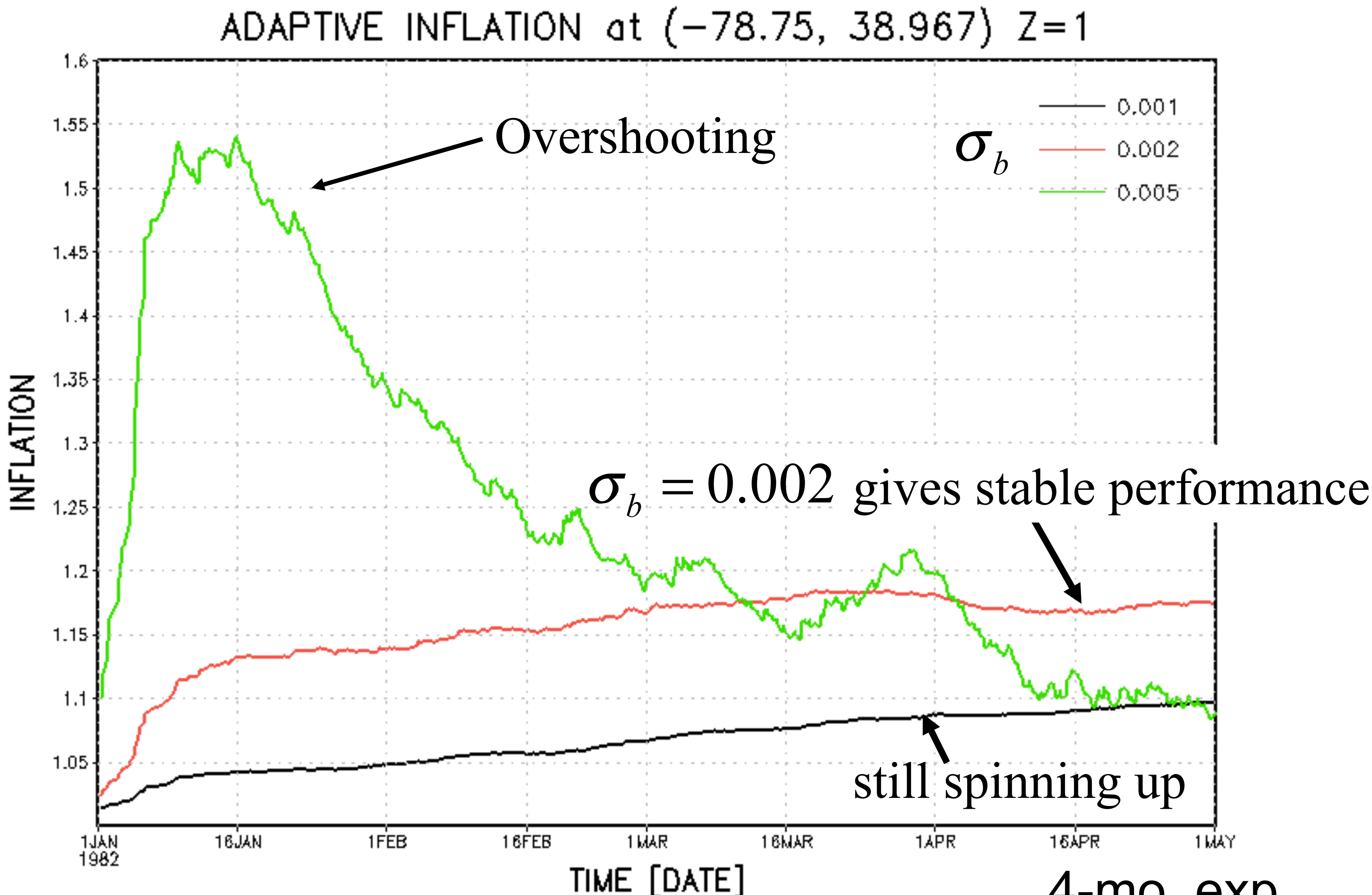
Results with the SPEEDY model

Estimated adaptive inflation



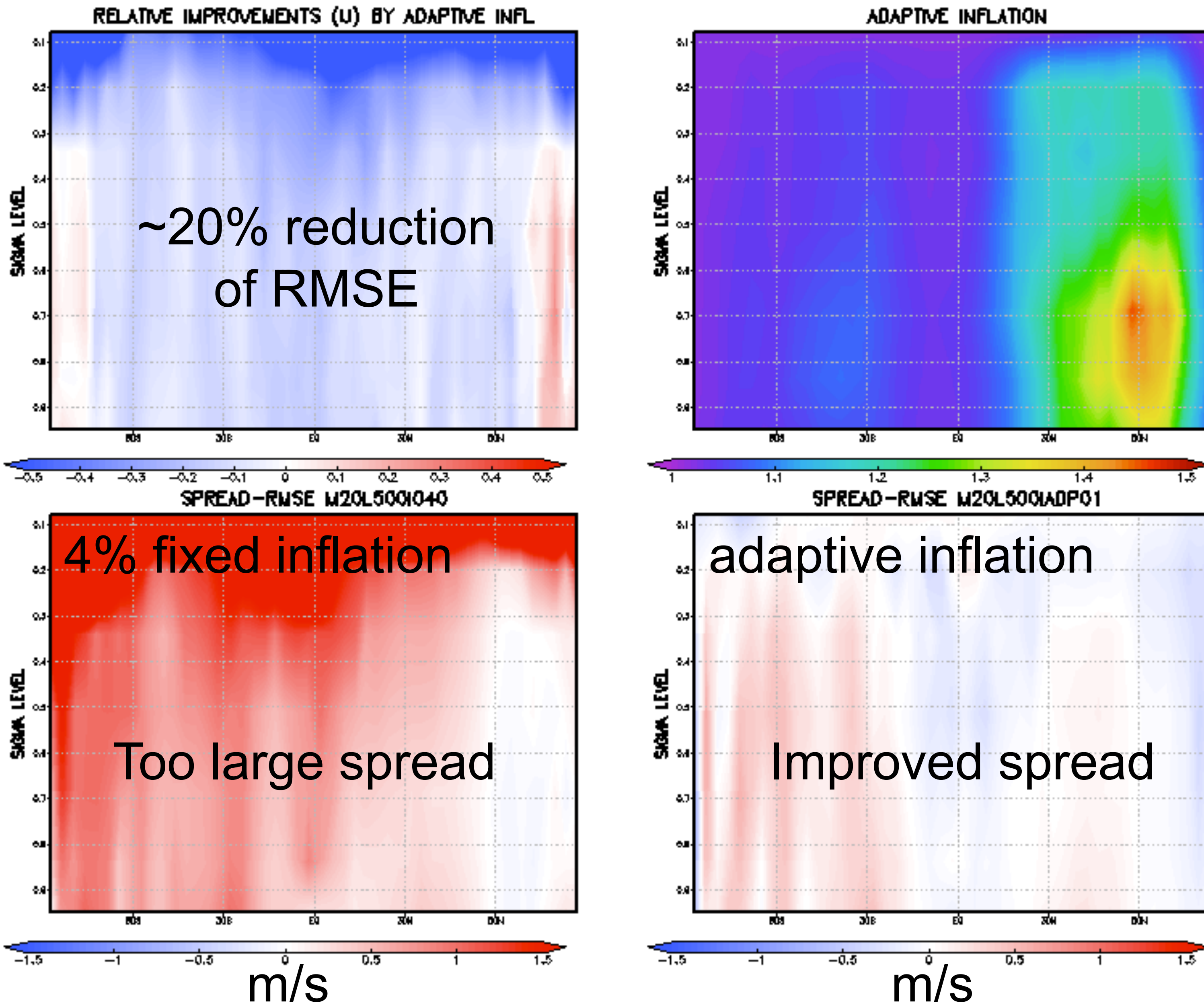


Time series of adaptive inflation

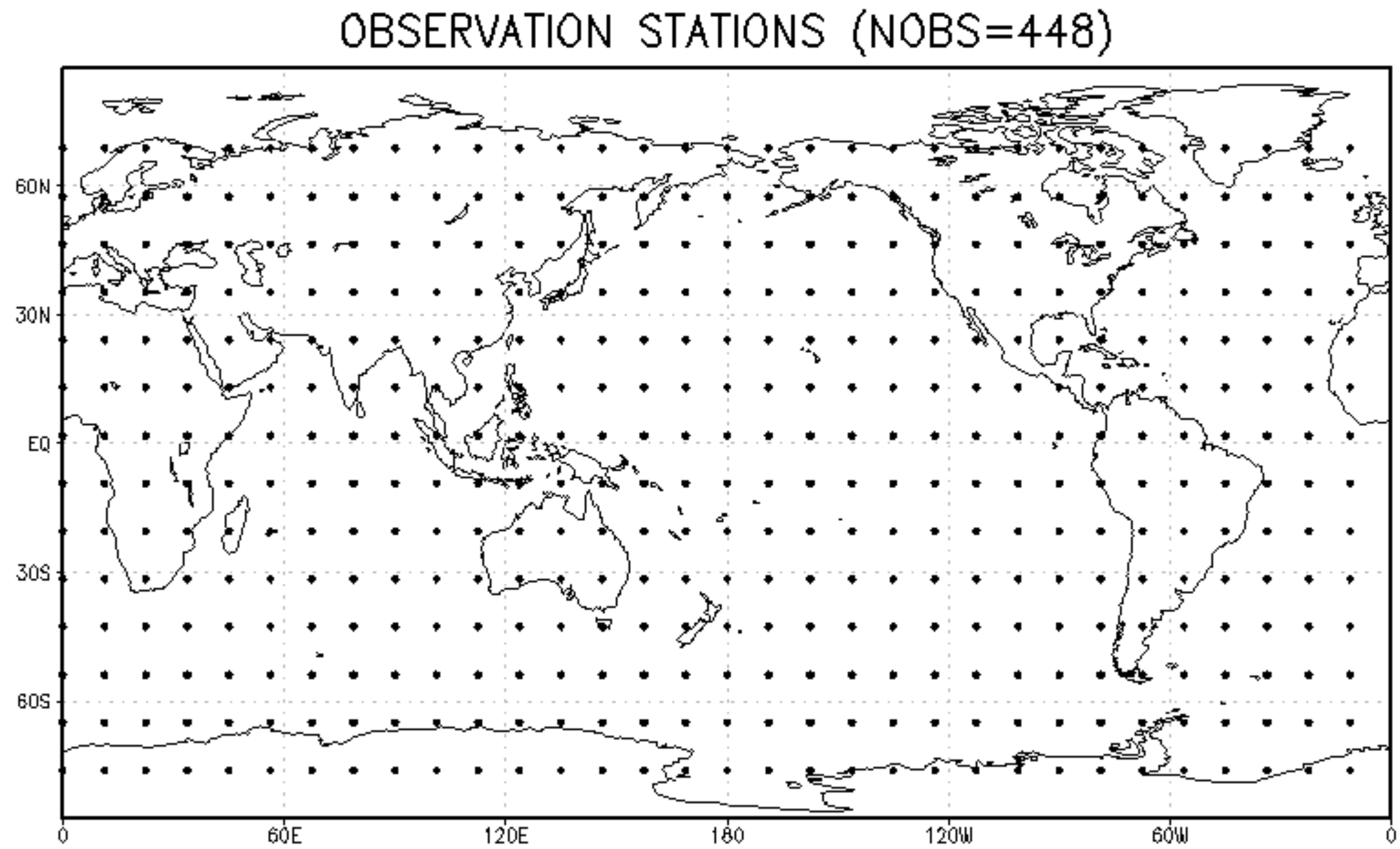


4-mo. exp.

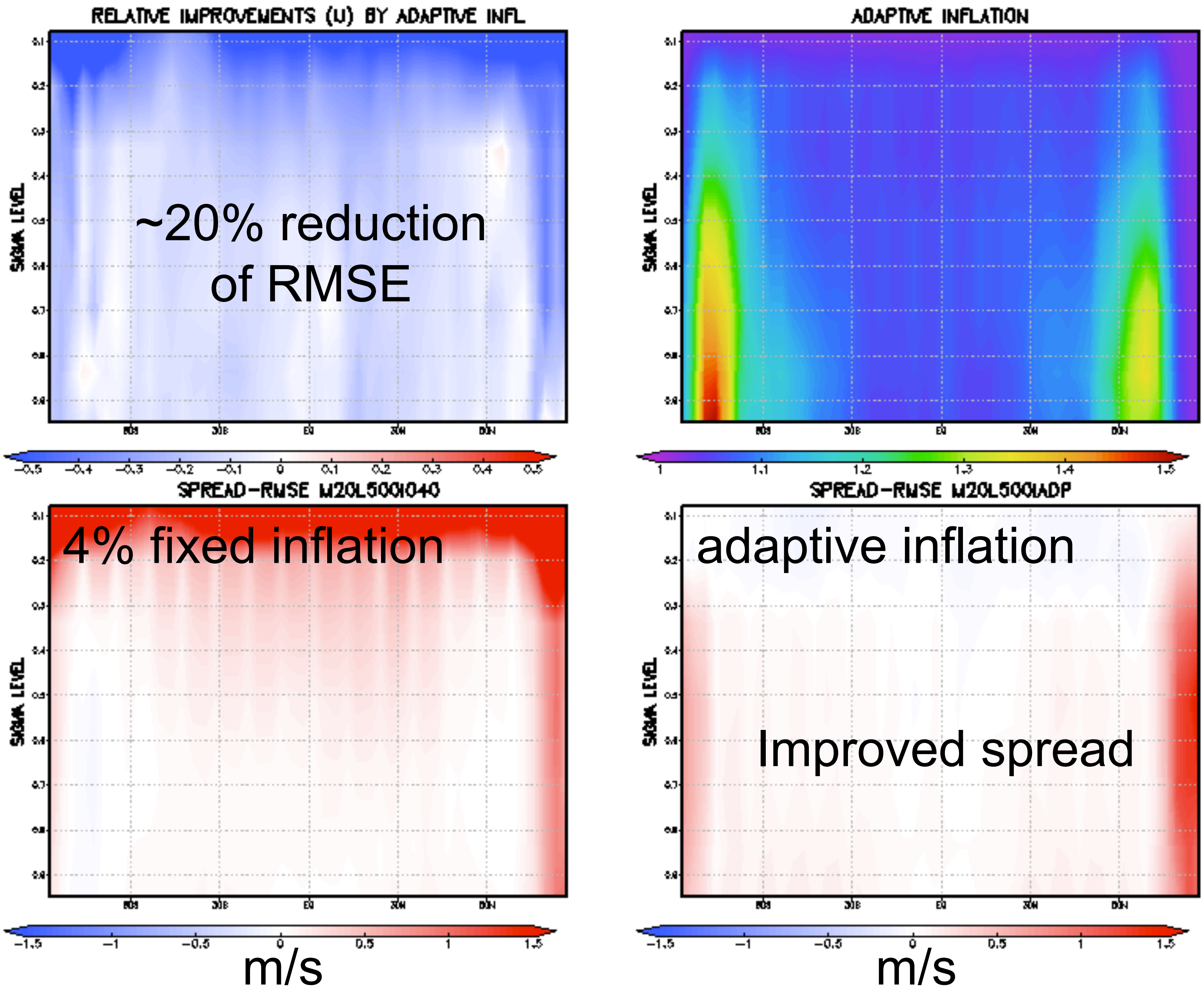
Averages of the final 1 mo. of 4-mo. experiments



Regular obs network



With regular obs network



Conclusion

- The proposed adaptive inflation method was tested with the Lorenz-96 and SPEEDY models
  - stable performance
  - improved analysis accuracy/ensemble spread

- Significant sensitivity with the choice of σ_b

Future work

- Application to other systems
  - Realistic NWP models
  - Ocean models
  - Martian atmosphere models

- Assimilation of real observations
  - Model errors
  - Temporally varying observing network
    - (e.g., aircraft, satellites)